Optimization of Separated Spacecraft Interferometer Trajectories in the Absence of A Gravity-Well

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Objective & Approach

Objective : ***Determine the optimal synthetic imaging trajectory for a Separated Spacecraft Interferometer***

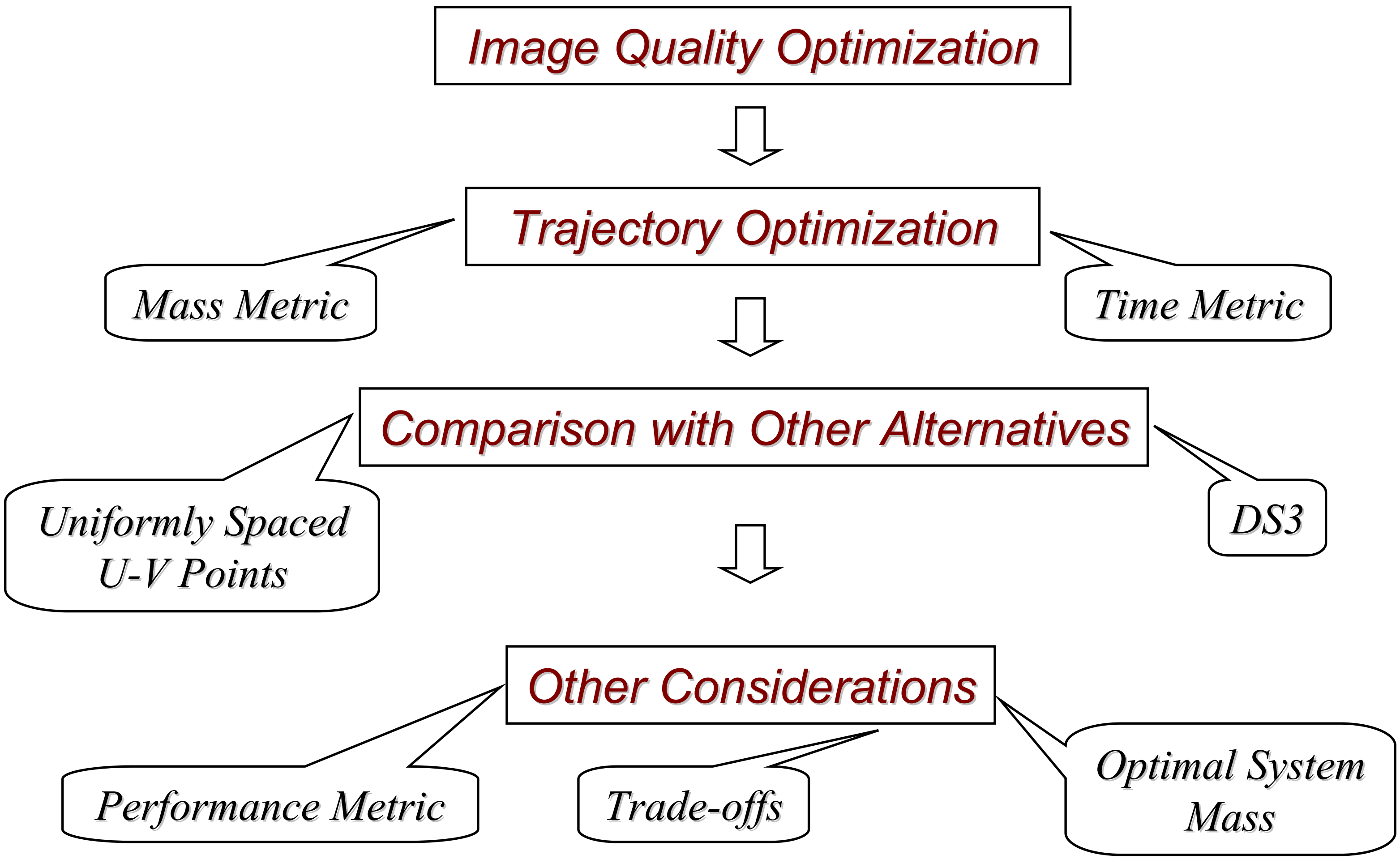


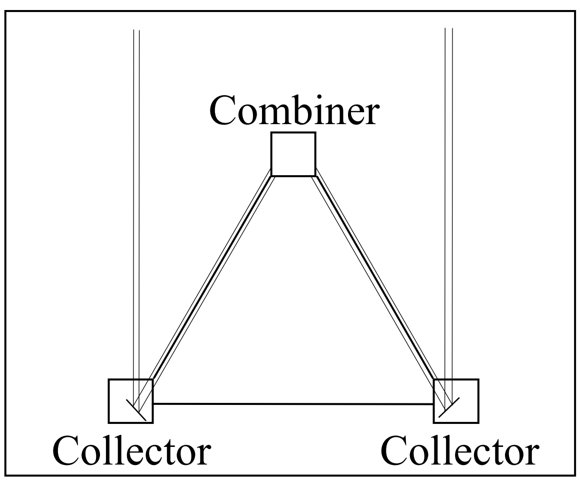
Image Quality

Model **: 2 Collector and 1 Combiner Interferometer (DS 3)**

Physics : Average Image Intensity

$$\bar{I}(\varphi_i,\varphi_j) = \frac{1}{N}\sum_{k=1}^{N}\left(\frac{\pi(1+\cos\theta)D}{\lambda}\right)^2\left(\frac{J_1\left(\frac{\pi D\sin\theta}{\lambda}\right)}{\frac{\pi D\sin\theta}{\lambda}}\right)^2\left(2\cos\frac{\pi}{\lambda}(\varphi_i x_k+\varphi_j y_k)\right)^2$$

$$\approx \frac{1}{N}\sum_{k=1}^{N}\left(\frac{2\pi D}{\lambda}\right)^2\left(2\cos\frac{\pi}{\lambda}(\varphi_i x_k+\varphi_j y_k)\right)^2$$



Nominal Point Spread Function (2601 U-V Points)

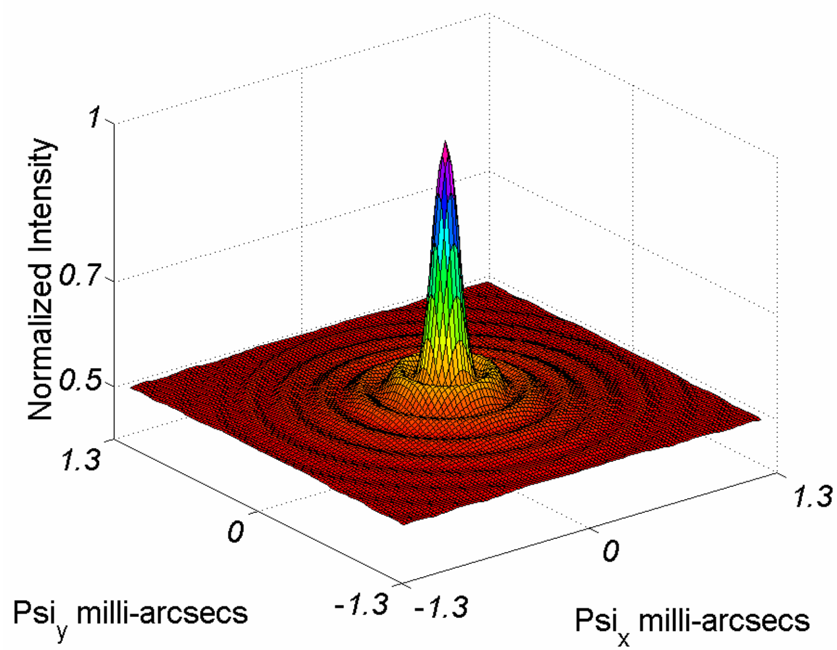


Image Quality - Mean Square Error

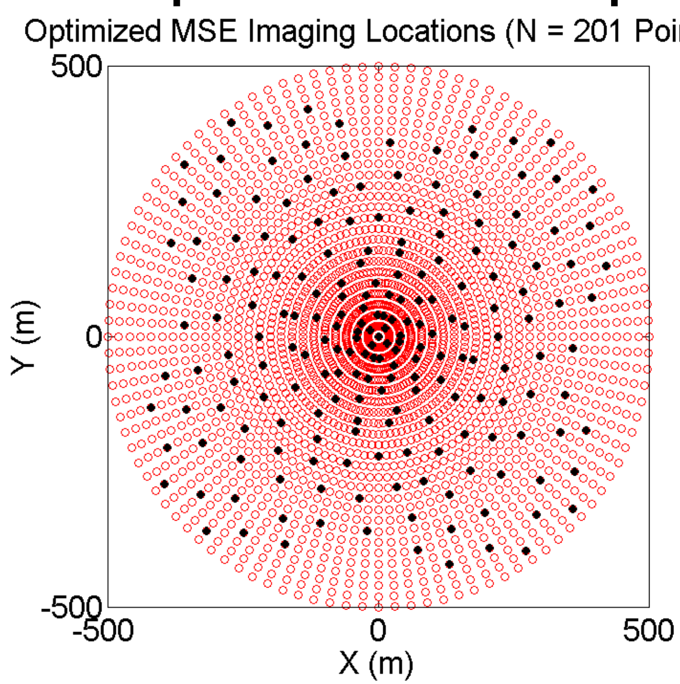
Minimize **:**

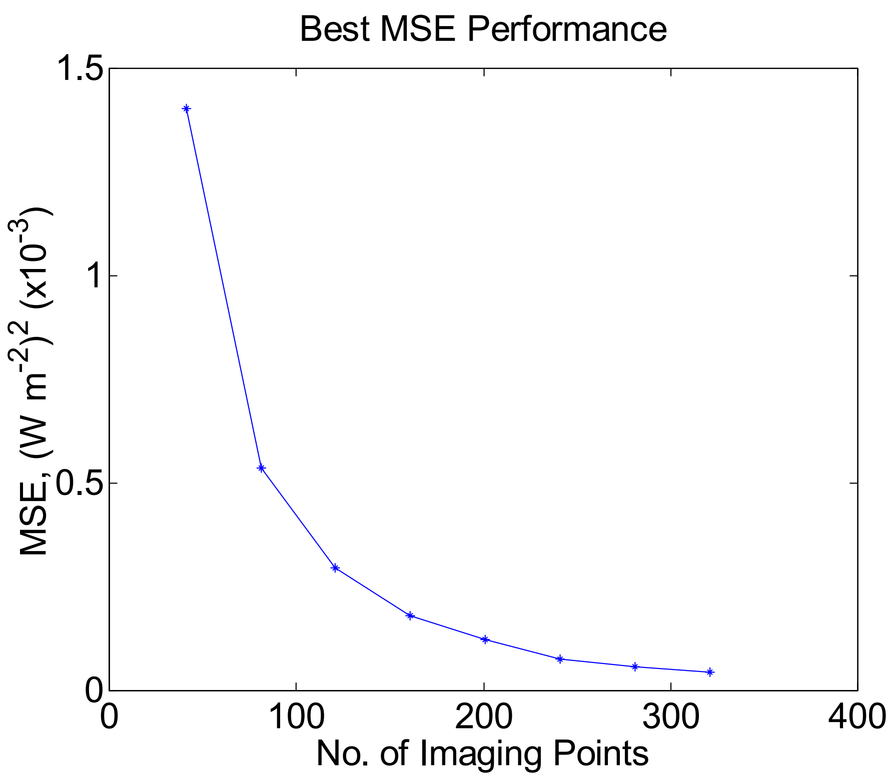
$$MSE = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m}\left(\bar{I}_{(\varphi_i,\varphi_j)} - \bar{I}_{o(\varphi_i,\varphi_j)}\right)^2}{m \times m}$$

Approach **: Choose N subset of all points**

: Compare with Nominal PSF

: Simulated Annealing Optimization Technique

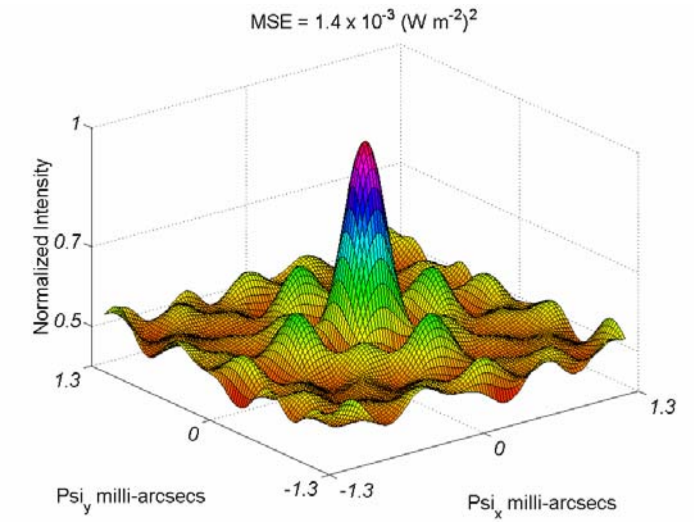




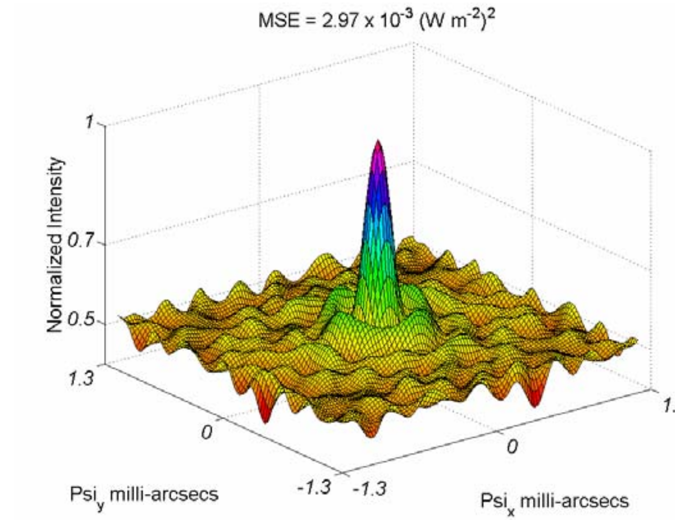
Results **: Image quality increases with no. of imaging points (N)**

: Diminishing rate of return

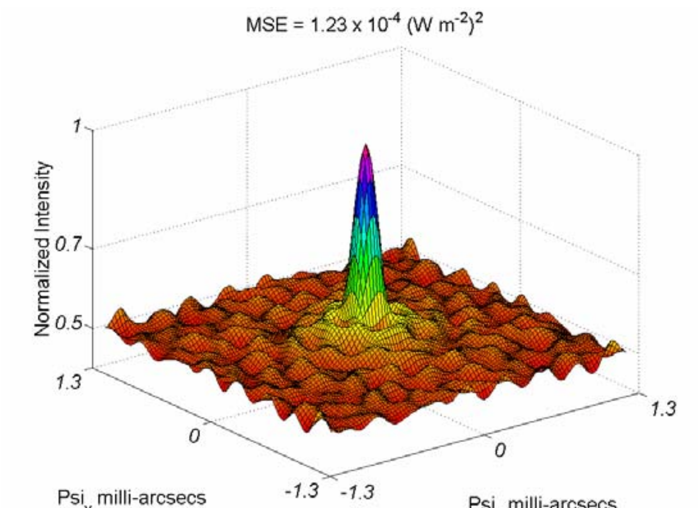
Point Spread Function Images



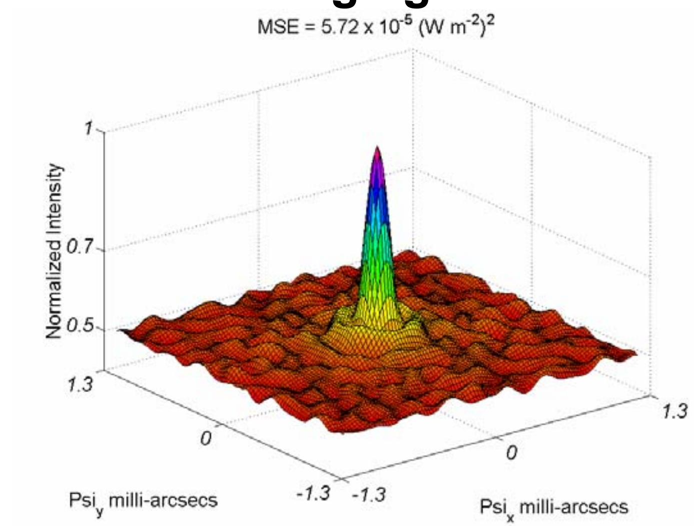
41 Imaging Points



121 Imaging Points



201 Imaging Points



281 Imaging Points

Trajectory Optimization - Mass Metric

Minimize : $m_{fuel} = \frac{\dot{m}_{fuel}}{2}\left(T_{image} \pm \sqrt{T_{image}^2 - 4\frac{N}{a}\sum_{i=1}^{N} s_i}\right)$

Assumptions : **"Stop and Stare" imaging mode**

: **Trapezoidal velocity profile**

: **Constant acceleration**

Parameters : **Spacecraft masses**

Collector = 150 kg

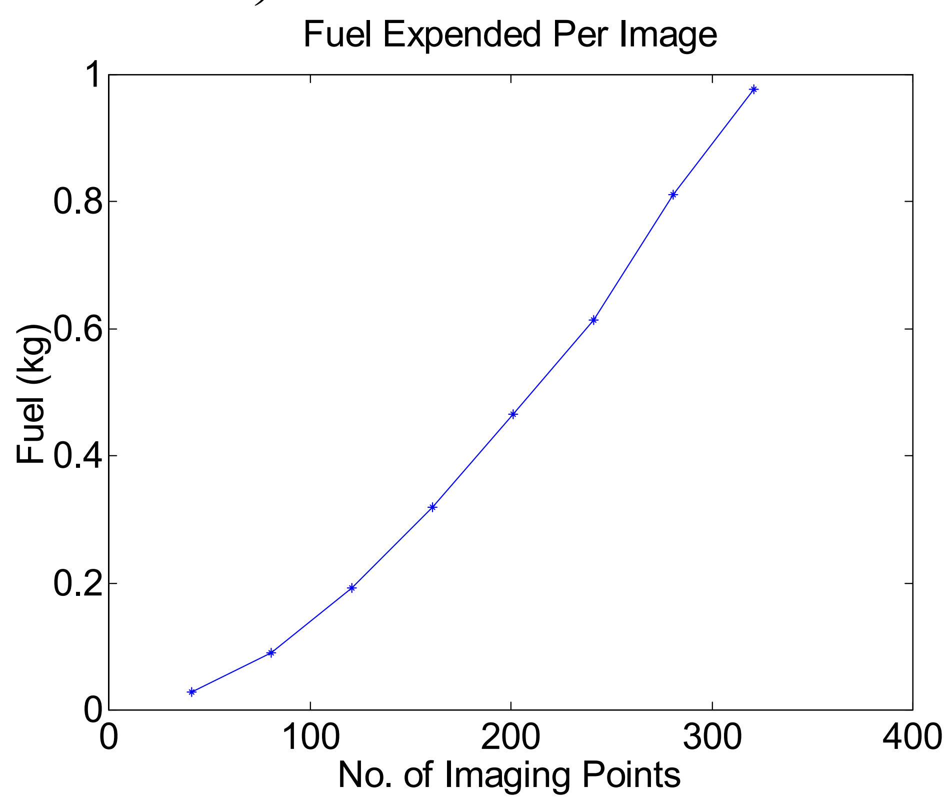
Combiner = 250 kg

: **Cold Gas Propulsion**

I_{sp} = 62.5 s, F = 9 mN

Constraint : **T_{image} = 264 Hours**

Approach : **Traveling Salesman Algorithm**



Result : **Fuel mass increases with no. of imaging points (N)**

Trajectory Optimization - Time Metric

Minimize : $$T = \frac{2}{\sqrt{a}} \sum_{n=1}^{N} \sqrt{s_i}$$

Assumptions : **"Stop and Stare" imaging mode**

: **Triangular velocity profile**

: **Small Integration Time**

Parameters : **Spacecraft masses**

Collector = 150 kg

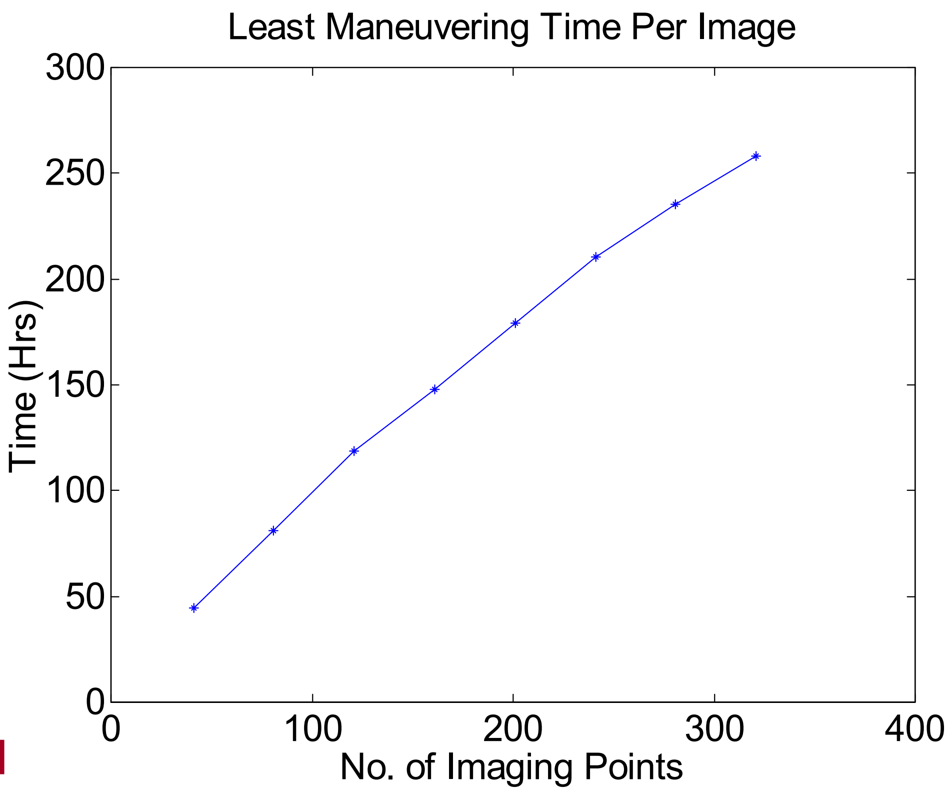
Combiner = 250 kg

: **Pulse Plasma Thrusters**

I_{sp} = 1000 s, F = 1.4 mN

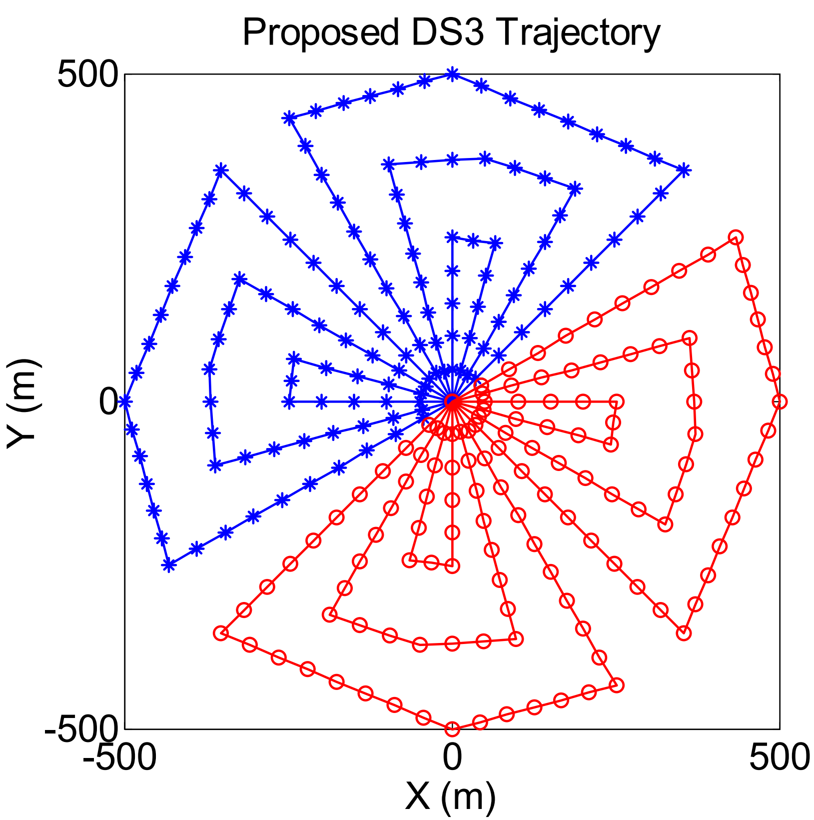
Constraint : **S/C Power 80 W**

Approach : **Traveling Salesman Algorithm**



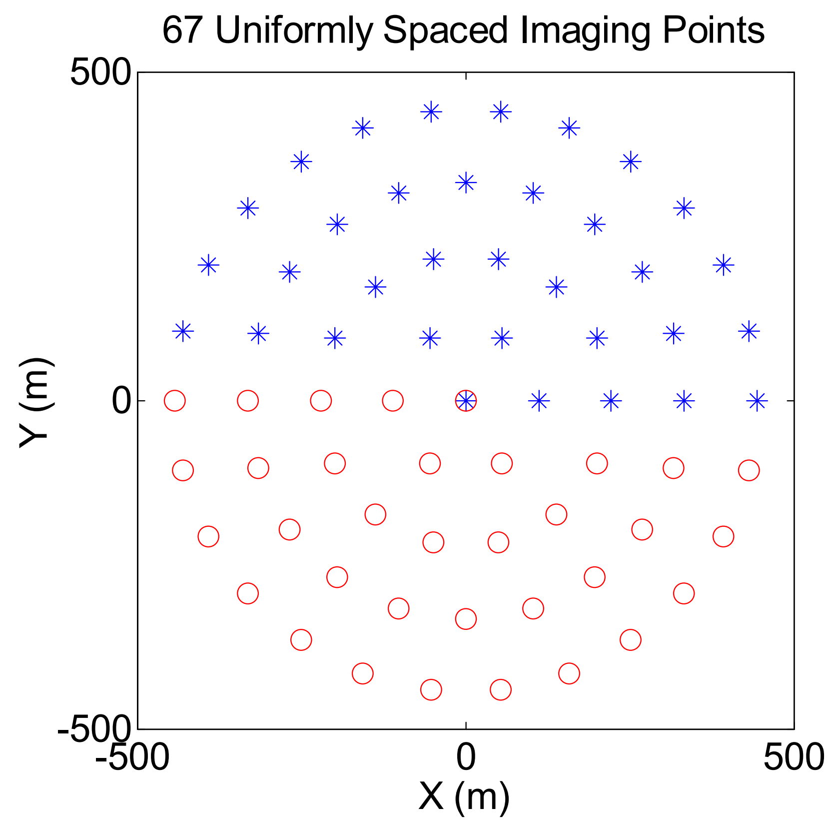
Result : **Imaging time increases with no. of imaging points**

Other Alternatives



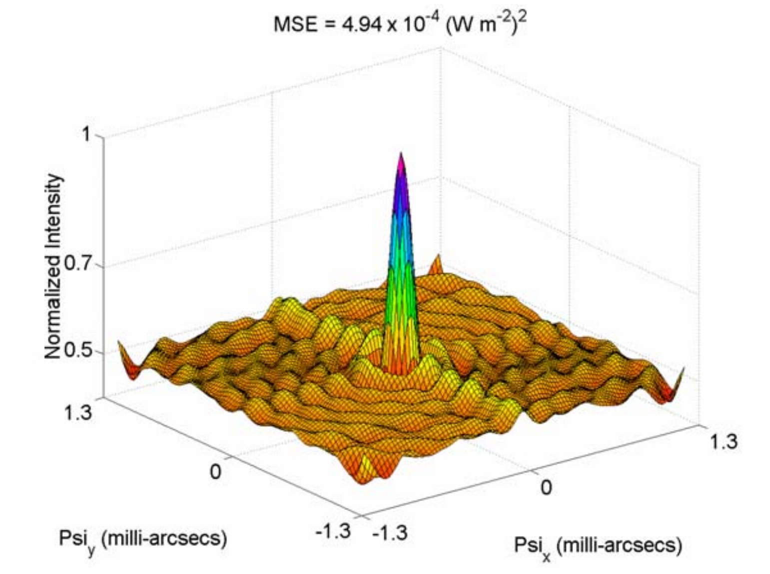
Proposed DS3

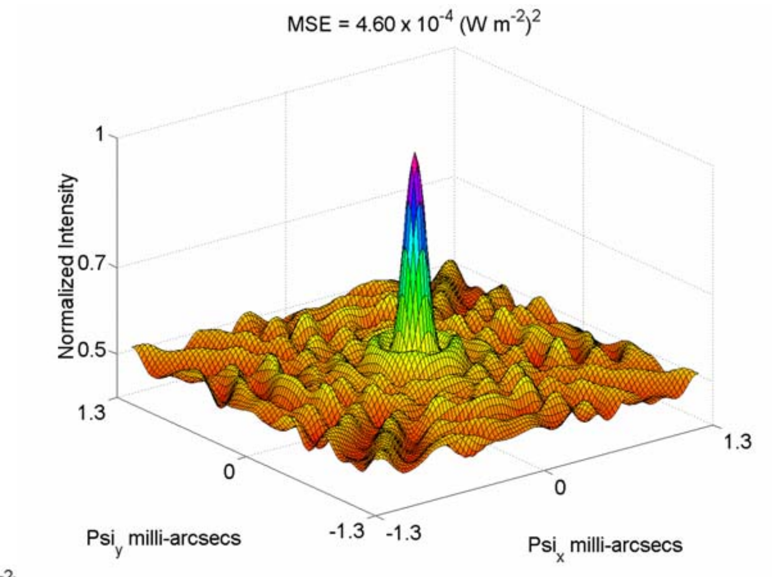
Reference : Linfield, JPL



Uniformly Spaced

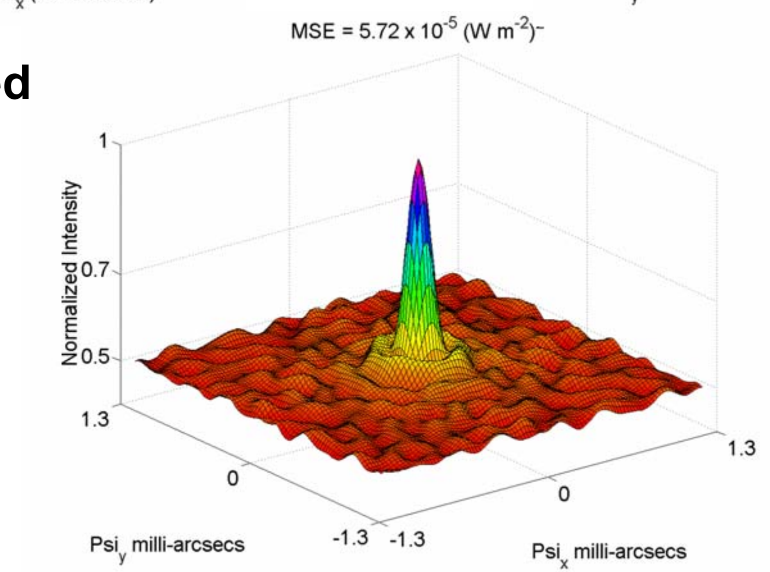
PSF Comparison





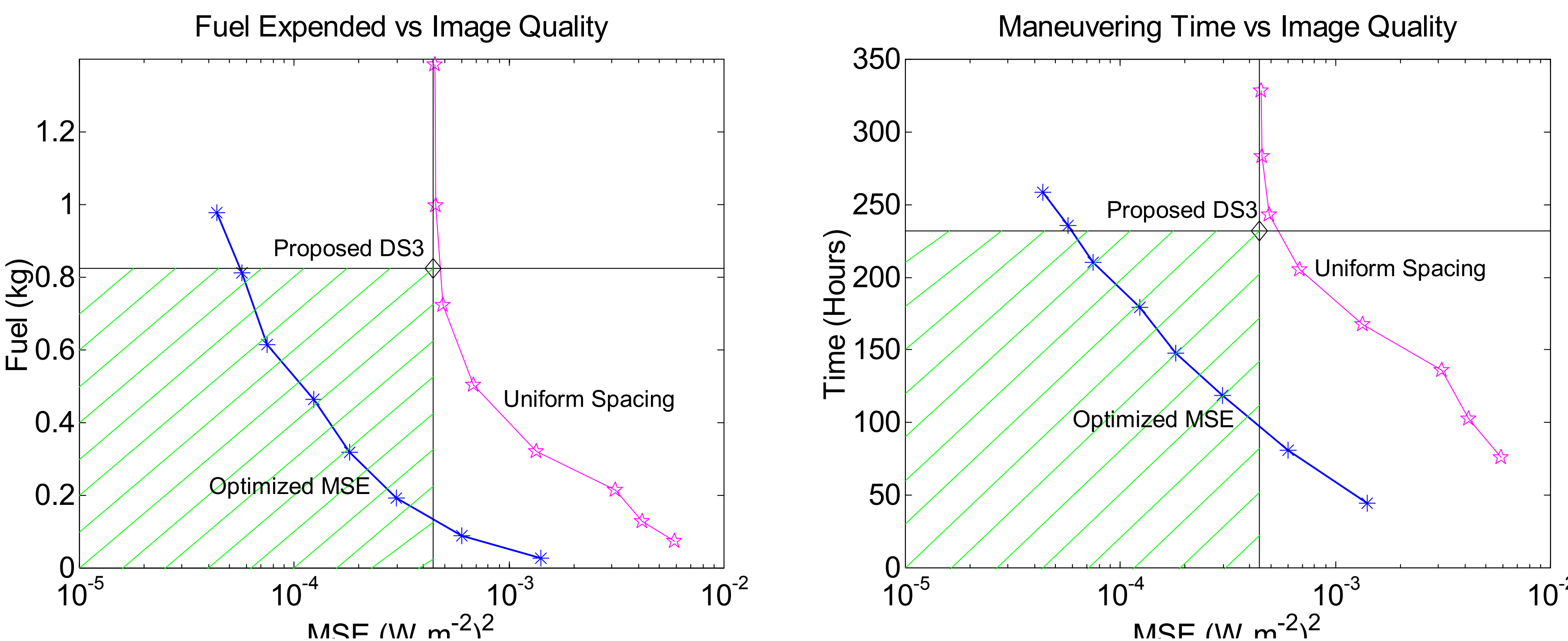
231 Uniformly Spaced Imaging Points

Proposed DS3 (261 Points)



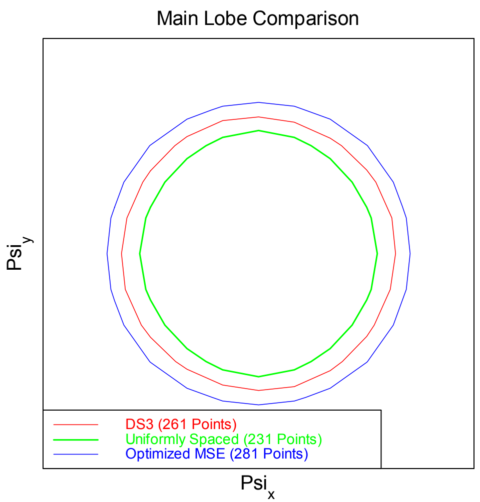
281 Optimal MSE Imaging Points

Fuel and Time Metrics vs MSE

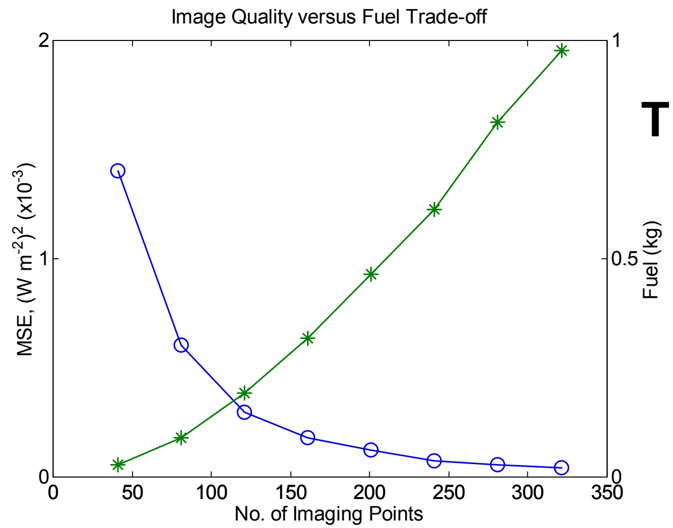


Result **: Better MSE with lower fuel consumption or shorter imaging time**

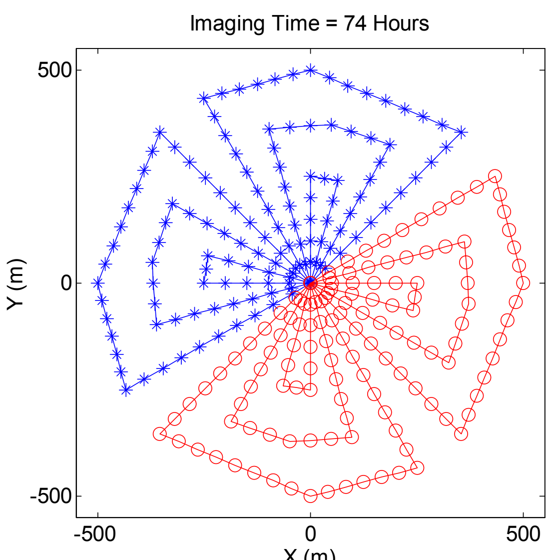
Other Considerations



Main Lobe Comparison

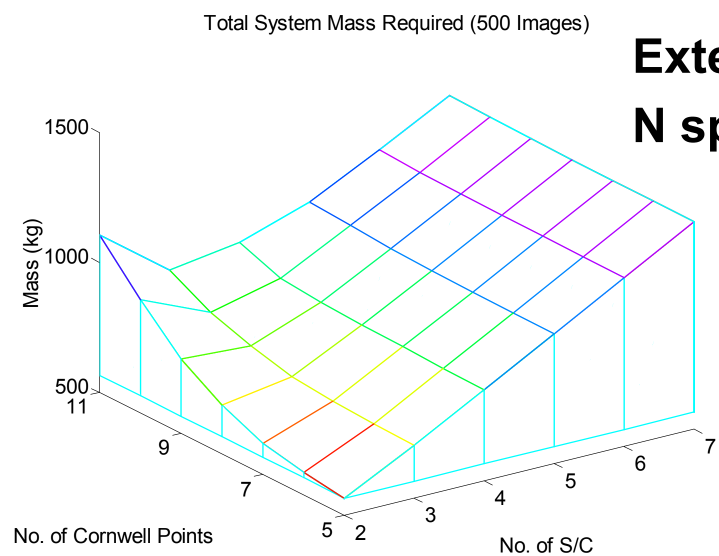


Trade-off



"Image on the Fly"

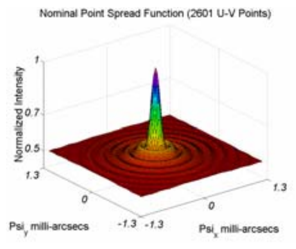
Reference : Linfield, JPL



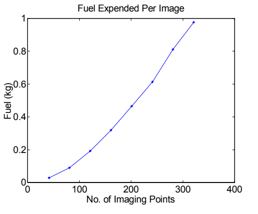
Extension to N spacecraft

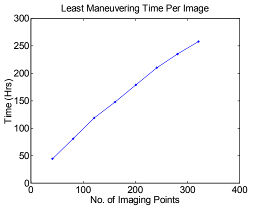
Conclusion

- Determined the optimal imaging locations

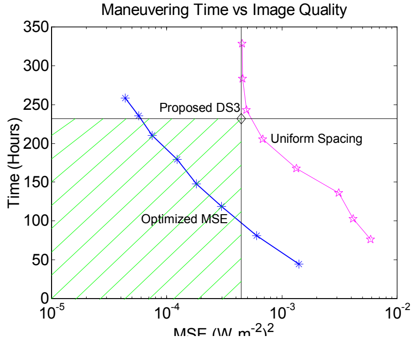


- Determined the optimal trajectories
  - Mass metric
  - Time metric

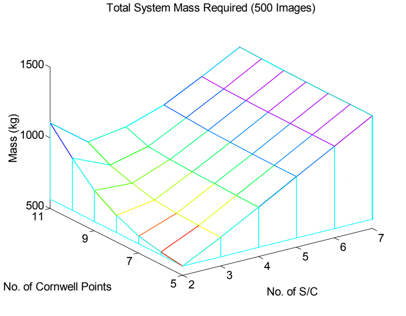




- Compared with other alternatives



- Future considerations
  - MSE versus Mass trade-off
  - MSE versus Time trade-off
  - Extension to N spacecraft
  - "Image on the Fly" mode
  - Other Metrics



Simulated Annealing

- Statistical Approach
- Randomly select a configuration and calculate cost, C_r
  - If $C_r < C_{r-1}$ ⇨ accept r^{th} configuration
  - If $C_r > C_{r-1}$ ⇨ accept r only if $\exp(-C_r/T) > \text{Random}(0,1)$
  - when C_r is accepted, decrease T (system temperature)
  - Continue until system is frozen (no new solution accepted in N trials)
- Does not guarantee global minimum
- Quick and easy implementation
- Reasonable solution achieved in short computation time
- Reference:
  - S. Kirkpatrick, C. D., Gelatt, Jr., M. P. Vecchi, “Optimization by Simulated Annealing”, Science, Volume 220, Number 4598, 13th May 1983.