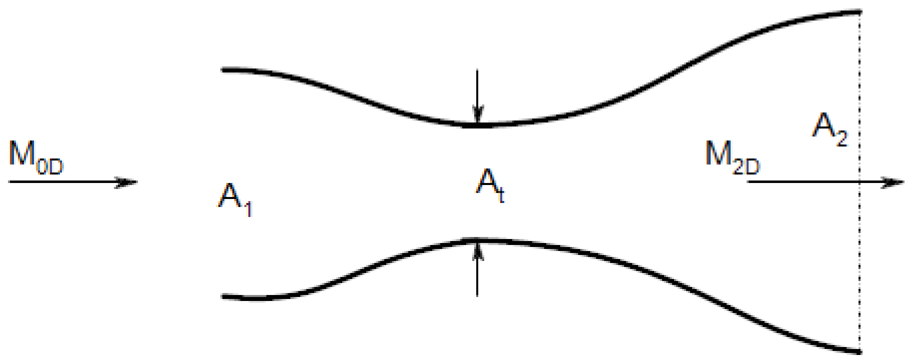
# Homework 8: Supersonic Internal-Compression Inlet

**a)** ***Geometry for the design condition:***




$M_0 = M_{0D} = 1.5$
$M_2 = M_{2D} = 0.5$

**Apply the continuity equation between stations 1 and 2:**

$$\frac{A_1}{A_2} = \frac{M_2}{M_0}\left(\frac{1+\frac{\gamma-1}{2}M_0^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (1)$$

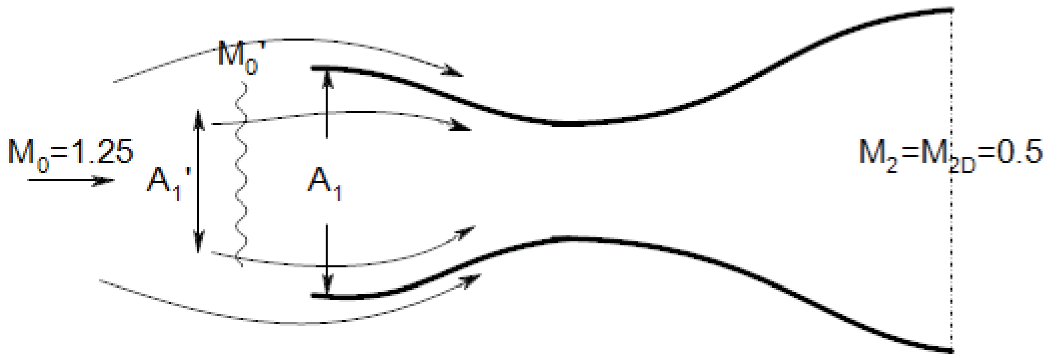
$$\frac{A_1}{A_2} = \frac{0.5}{1.5}\left(\frac{1+0.2\times 2.25}{1+0.2\times 0.25}\right)^3 = 0.87784$$

**Similarly, apply continuity between stations t and 2 (with $M_t = 1$):**

$$\frac{A_t}{A_2} = \frac{M_2}{M_t}\left(\frac{1+\frac{\gamma-1}{2}M_t^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = \frac{0.5}{1}\left(\frac{1+0.2}{1+0.2\times 0.25}\right)^3 = 0.74636$$

b) Now, with this geometry, consider $M_0 = 1.25 < M_{0D}$.

**A shock appears in the incoming flow:**




**Behind the shock:**

$$M_0' = \sqrt{\frac{1+\frac{\gamma-1}{2}M_0^2}{\gamma M_0^2 - \frac{\gamma-1}{2}}} = \sqrt{\frac{1+0.2\times1.25^2}{1.4\times1.25^2-0.2}} \qquad (2)$$

$$M_0' = 0.81264$$

**The total pressure ratio across the shock is:**

$$\frac{P_{t0}'}{P_{t0}} = \left(\frac{\gamma+1}{2\gamma M_0^2-\gamma-1}\right)^{\frac{1}{\gamma-1}}\left(\frac{(\gamma+1)M_0^2}{(\gamma-1)M_0^3+2}\right)^{\frac{\gamma}{\gamma-1}} \qquad (3)$$

$$\frac{P_{t0}'}{P_{t0}} = \left(\frac{2.4}{2.8\times1.25^2-0.2}\right)^{2.5}\left(\frac{2.4\times1.25^2}{0.4\times1.25^2+2}\right)^{3.5} = 0.98706$$

Starting right after the shock, the flow area that is captured by the engine is $A_1' < A_1$. The flow is insentropic and subsonic between there and station 2.

**Apply continuity again, this time between 1' and 2:**

$$\frac{A_1'}{A_2} = \frac{M_2}{M_0'}\left(\frac{1+\frac{\gamma-1}{2}M_0'^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = \frac{0.5}{0.81264}\left(\frac{1+0.2\times0.81264^2}{1+0.2\times0.25}\right)^3 = 0.77114$$

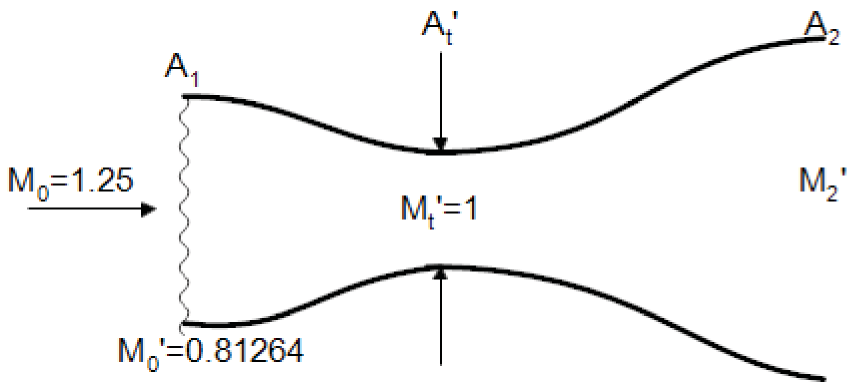
**Dividing by $\frac{A_1}{A_2} = 0.87784$:**

$$\frac{A_1'}{A_1} = 0.87845$$

**The *spillage fraction* is then:**

$$1 - 0.87845 = 0.12155$$

c) The control system now increases the throat area to $A_t' > A_t$, while simultaneously allowing$M_2$ to increase to $M_2'$ to accommodate the extra flow (without spillage now). The new situation looks like this: The shock is right at the lip and the throat is just choked ($M_t' = 1$, with $M < 1$ on both sides of the throat).




**Continuity from throat to station 2 and between station 1 and the throat:**

$$\frac{A_t'}{A_2} = \frac{M_2'}{1}\left(\frac{1+\frac{\gamma-1}{2}}{1+\frac{\gamma-1}{2}M_2'}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (4)$$

$$\frac{A_t'}{A_1} = M_0'\left(\frac{1+\frac{\gamma-1}{2}}{1+\frac{\gamma-1}{2}M_0'}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (5)$$

**From equation (5):**

$$\frac{A_t'}{A_1} = 0.81264\left(\frac{1.2}{1+0.2\times0.81264^2}\right)^3 = 0.96786$$

$$\frac{A_t'}{A_1} = 0.96786 \times 0.87784 = 0.84963$$

(compared to $\frac{A_t}{A_2} = 0.74636$)

**Using this in equation (4):**

$$0.84963 = M_2'\left(\frac{1.2}{1+0.2M_2'^2}\right)^3 \qquad (6)$$

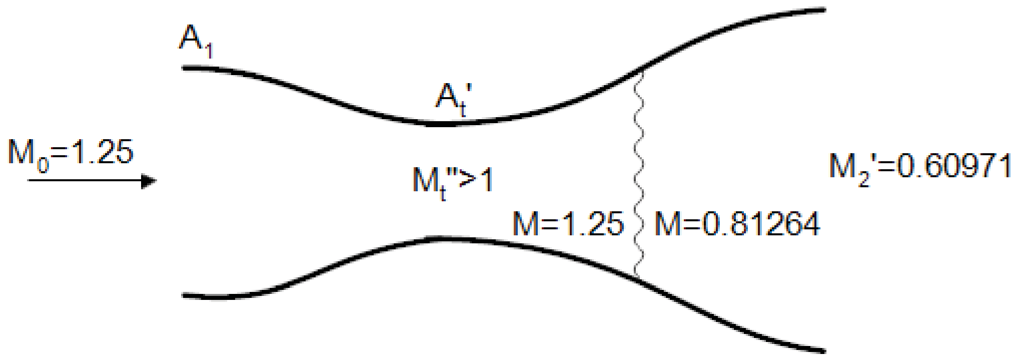
**Iterating:**

$M_2' = 0.60971$

(compared to $M_2 = 0.5$)

Since the normal shock is still at $M_0 = 1.25$, the total pressure ration is still 0.98706m but the engine is taking in 12% more flow.

**d)** The shock spontaneously pops into the location on the divergent section where $M = 1.25$. This ensures the same $P_t$ ratio, and the same (full) flow through the engine, with still the same $M_2 = 0.60971$.




**The new throat Mach number needs to satisfy continuity between station 1 and the throat:**

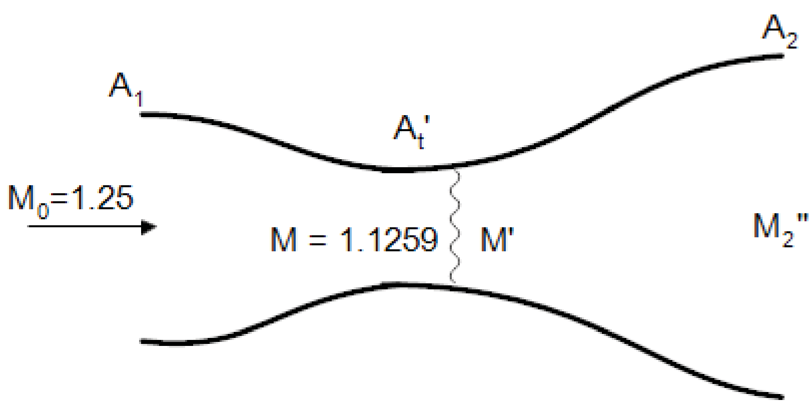
$$\frac{A_t'}{A_1} = \frac{M_0}{M_t''}\left(\frac{1+\frac{\gamma-1}{2}M_t''^2}{1+\frac{\gamma-1}{2}M_0^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$0.96787 = \frac{1.25}{M_t''}\left(\frac{1+0.2M_t''^2}{1+0.2\times1.25^2}\right)^3$$

**Iterating:**

$M_t'' = 1.1259$

**e)** Finally, the controller reduces $M_2$; since the flow remains unchanged (being the full "cookie-cutter" flow at $M_0$ and $A_1$), the total pressure behind the shock must be higher to compensate. This means lower shock losses, which is accomplished by the shock moving upstream to a Mach number lower than 1.25. In fact, the lowest one can get is the throat Mach number, 1.1259.




**Behind this newly located shock, the Mach number is:**

$$M' = \sqrt{\frac{1+0.2\times1.1259^2}{1.4\times1.1259^2-0.2}} = 0.89221$$

**The new $M_2 = M_2''$ must satisfy continuity behind the throat and station 2:**

$$\frac{A_t''}{A_2} = \frac{M_2''}{M'}\left(\frac{1+\frac{\gamma-1}{2}M'^2}{1+\frac{\gamma-1}{2}M_2''^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$0.84963 = \frac{M_2''}{0.89221}\left(\frac{1+0.2\times0.89221^2}{1+0.2M_2''^2}\right)^3$$

**Iterating:**

$M_2'' = 0.5994$

**The shock losses are now those for a shock at $M = 1.1259$:**

$$\frac{P_t'}{P_{t0}} = \left(\frac{2.4}{2.8\times1.1259^2-0.2}\right)^{2.5}\left(\frac{2.4\times1.1259^2}{0.4\times1.1259^2+0.2}\right)^{3.5} = 0.99796$$

***Note:*** Since the throat area is variable, this is not necessarily the final configuration at $M_0 = 1.25$. The pilot could now reduce $A_t$ until $M_t = 1$ is reached. The shock disappears and we have an *ideal inlet*, as if the design condition were actually $M_0 = 1.25$. Along with reducing $A_t$, $M_2$ would also have to be reduced, so as to pass the same flow with higher total pressure.

$M_t = 1$

$\frac{A_t}{A_2} = 0.83864$

$M_2 = 0.59646$

***Concept Questions***

**1)** In part c), while the throat area is being increased, the compressor inlet Mach number must also increase. From lecture 19, we know this can be accomplished by increasing $\theta = \frac{T_{t4}}{T_{t2}}$, or equivalently, increasing $f$. The compressor operating point moves up along the operating line, $\bar{m}_2$ increases ($M_2$ increases), and $\pi_c$ increases. So the pilot must *throttle up*.

**2)** How to continue on to $M_0 = 1.5$? One way to think of this is that the whole process of shock repositioning at $M_0 = 1.25$ could be simply deferred to $M_0 = 1.5$. This might not be feasible, however, because of the reduced thrust due to the strong shock losses and spillage at the higher $M_0$. Conceptually, *if the final step of reducing $A_t$ is also included*, we would end up with the ideal configuration of part a). Rather than doing it all at one $M_0$, we can start from the configuration of part c), reduce $A_t$ and $M_2$ according to the note in page 3, and then gradually continue to reduce $A_t$ and $M_2$ as $M_0$ increases, until the conditions of part a) are reached.

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