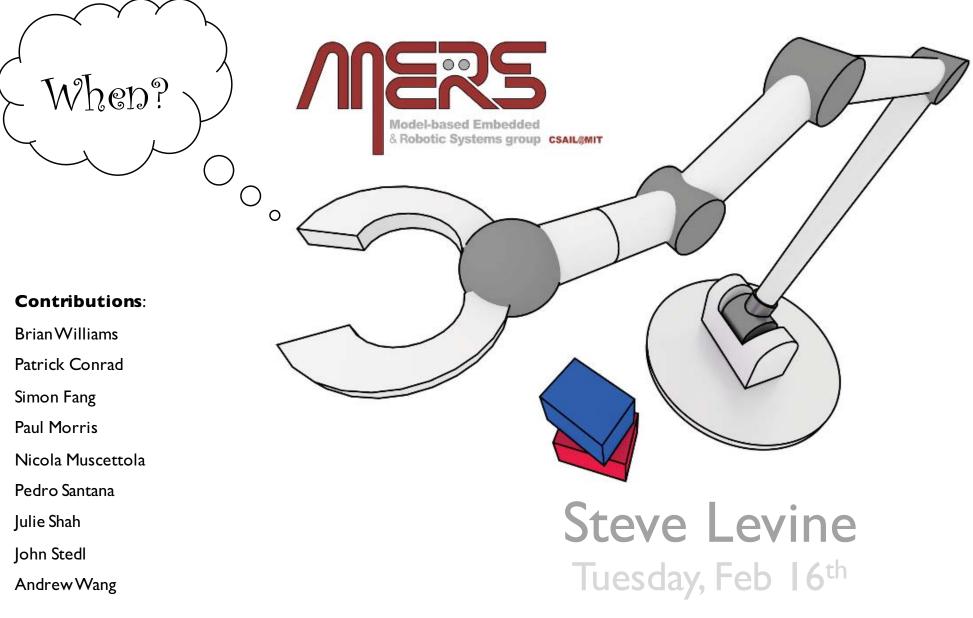




Programs with Flexible Time





Assignments



Problems Sets:

- Pset 1 due tomorrow (Wednesday) at 11:59pm
- Pset 2 released tomorrow

Interesting references:

- Dechter, R., I. Meiri, J. Pearl, "Temporal Constraint Networks," Artificial Intelligence, 49, pp. 61-95,1991.
- Muscettola, N., P. Morris and I. Tsamardinos, "Reformulating Temporal Plans for Efficient Execution." Intl Conf. on Knowledge Representation and Reasoning (KRR), 1998.



Outline



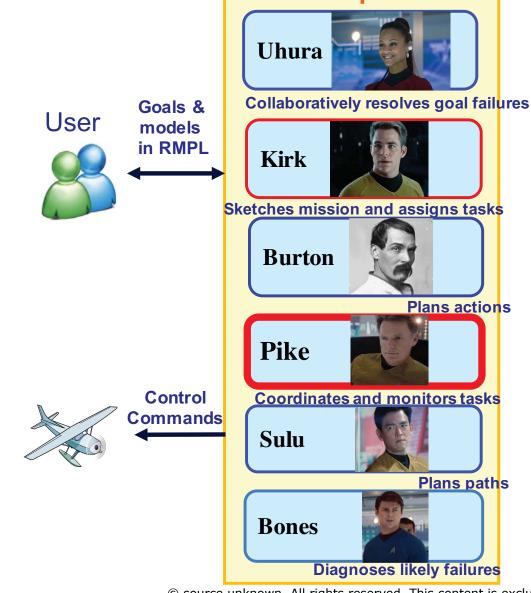
- Programs with Flexible Time
 - Intro
 - Describing temporal plans
 - Exposing implicit constraints
 - Consistency checking
 - Offline scheduling
 - Online execution
 - Reformulating for faster online execution



A single "cognitive system" language and executive.



Enterprise



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Flexible Time



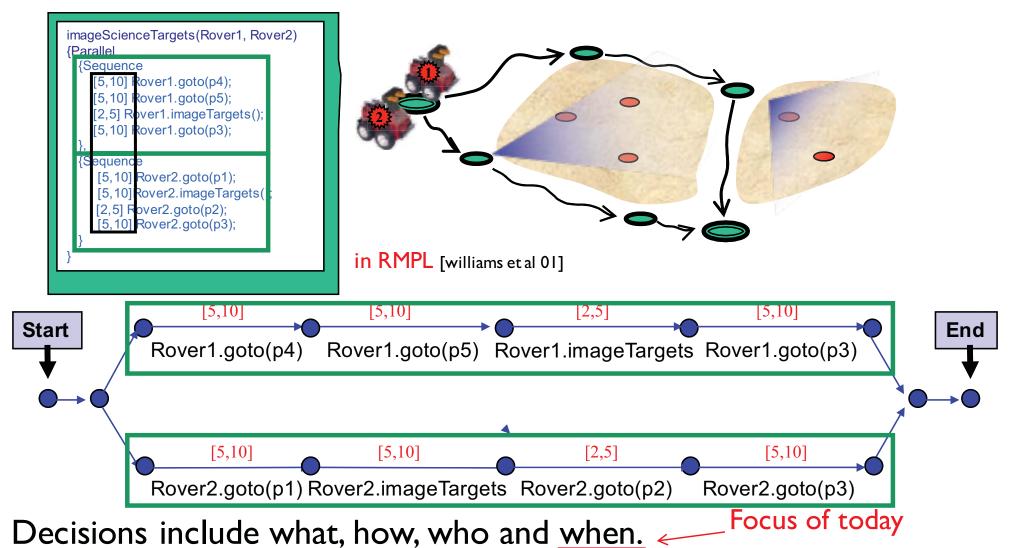
- Flexible time = more robustness
- We tell cognitive robot:
 - Timing requirements ("engage boosters 2-4 minutes after launch but before reaching orbit")
 - Cognitive robot schedules autonomously.



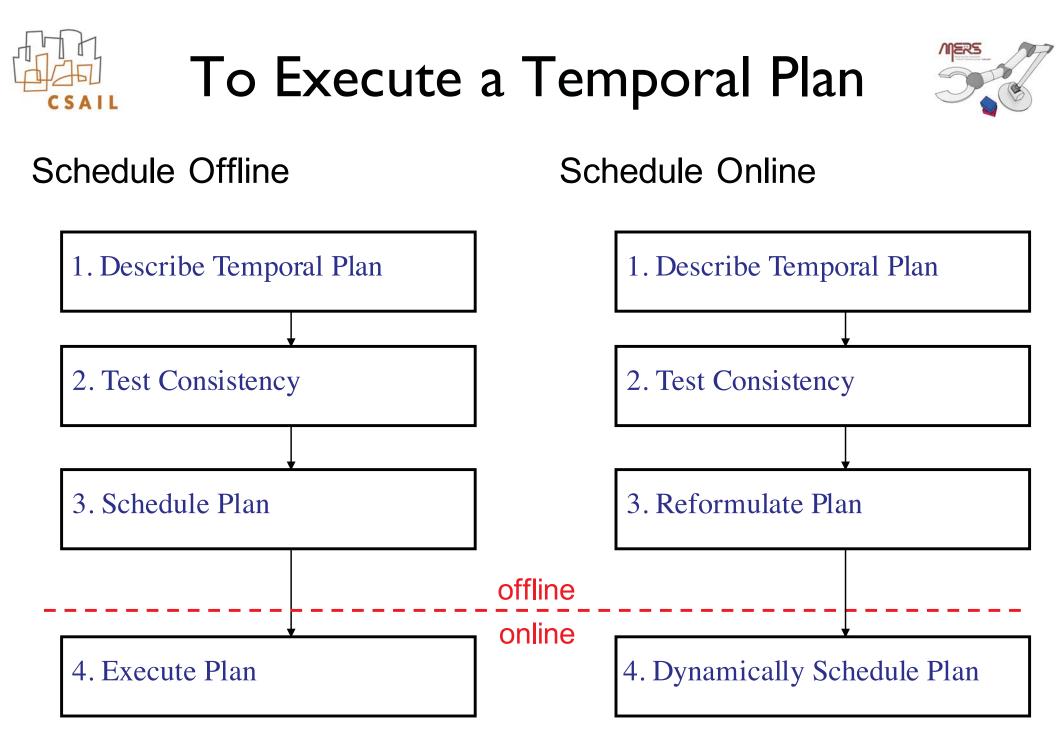


Execution of Timed Model-based Programs





Agents adapt to temporal disturbances in a coordinated manner by scheduling the start of activities on the fly. [Muscettola, Morris, Tsamardinos, KR 98]



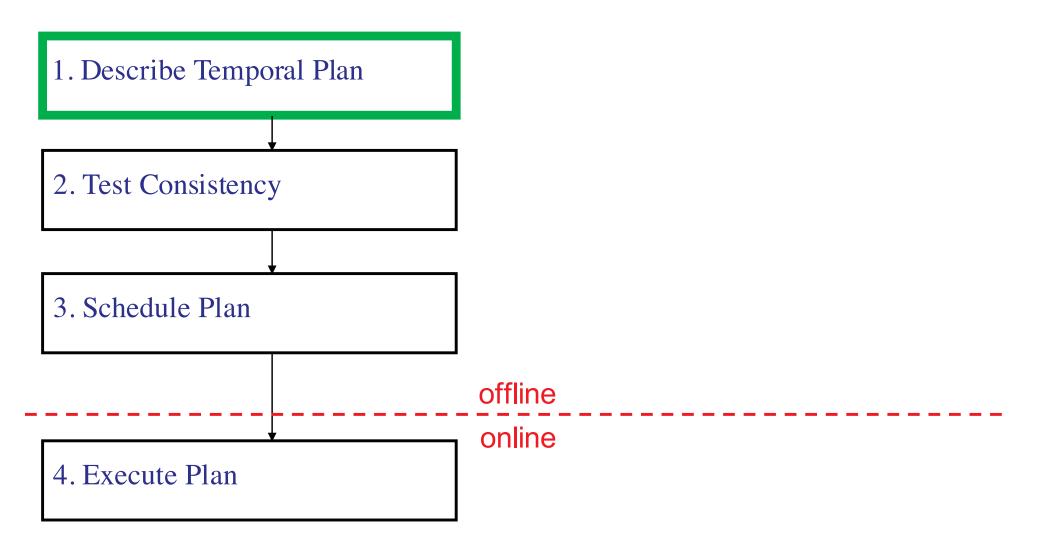


To Execute a Temporal Plan



Schedule Offline

Schedule Online





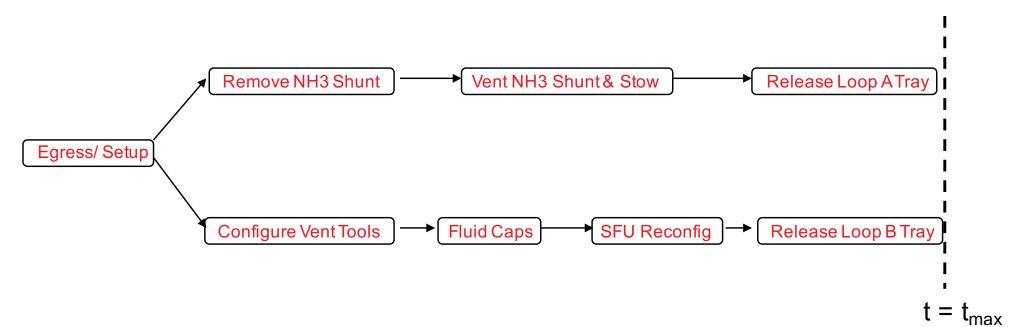
Describe Temporal Plan





This image is in the public domain.

- Activities to perform
- Relationships among activities





Metric Temporal Relations



• Going to the store takes at least 10 min and at most 30 min.

[10min, 30min]

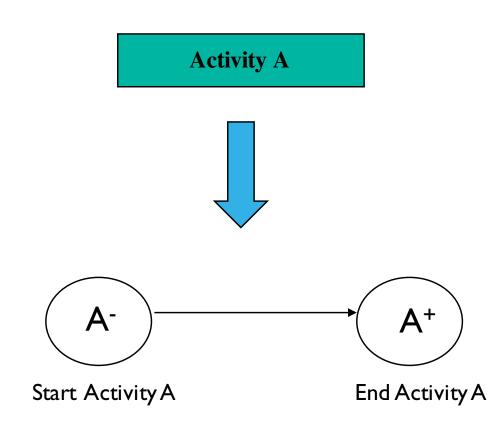
Activity: Going to the store

• Bread should be eaten within one day of baking.





Simplify by reducing interval relations to relations on timepoints.

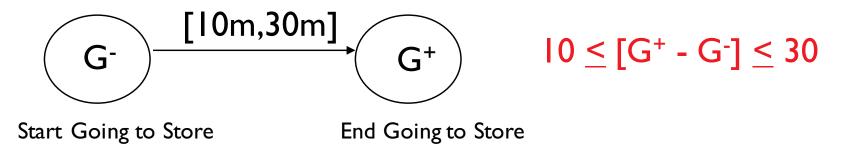




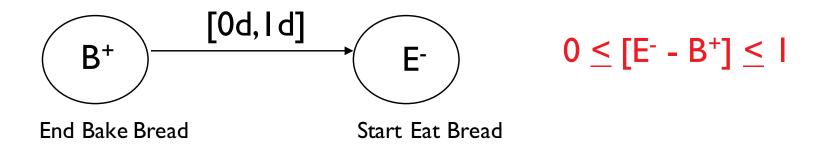
Metric Temporal Relations



• Going to the store takes at least 10 min and at most 30 min.

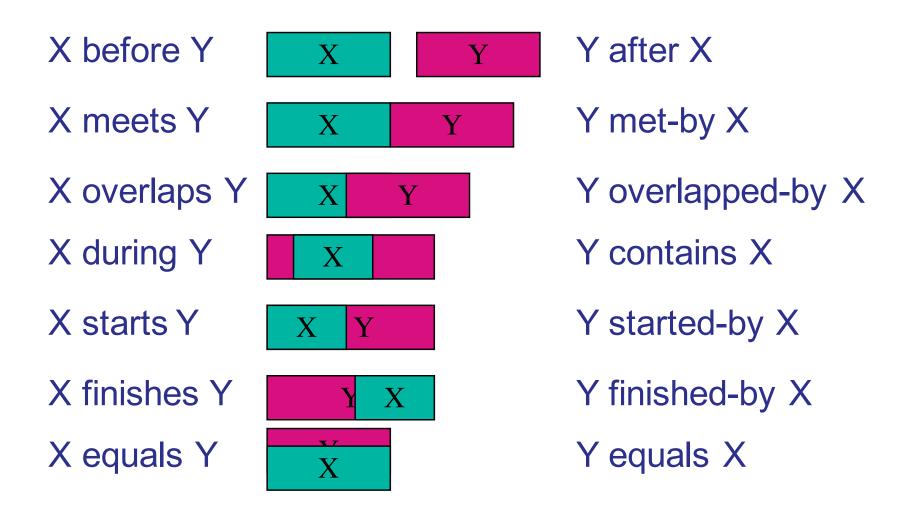


• Bread should be eaten within one day of baking.





[Allen 83]

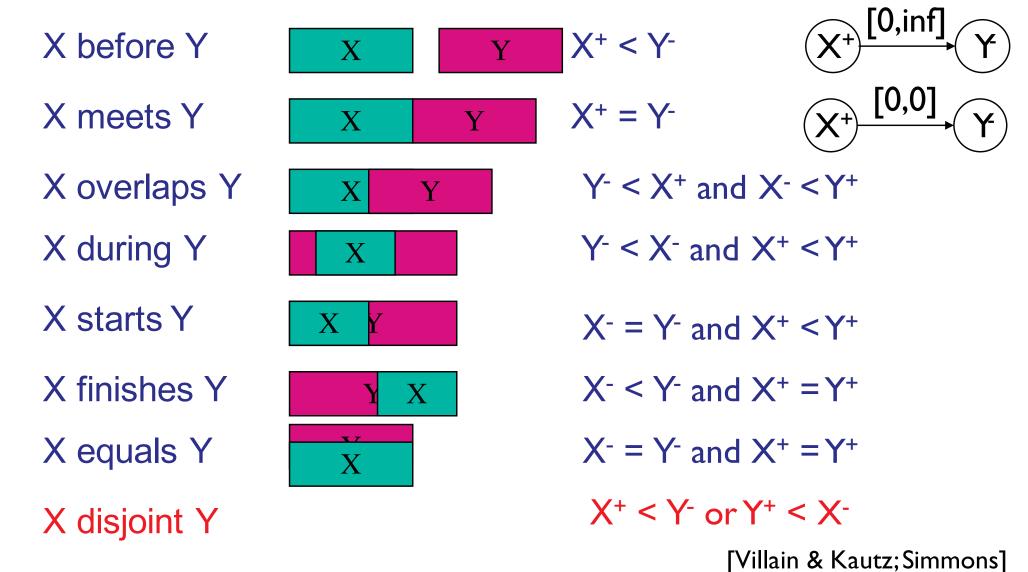


X disjoint Y





Expressed as timepoint inequalities:



Temporal Relations Described by a Simple Temporal Network (STN)

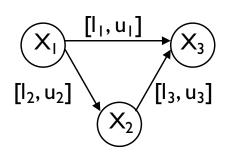
- Simple Temporal Network
 - Tuple <X, C> where:
 - variables X₁,...X_n, represent time points (real-valued domains)
 - binary constraints C of the form:

 $(X_k - X_i) \in [a_{ik}, b_{ik}]$

- called links.

Sufficient to represent: • simple metric constraints • all Allen relations but 1...

Can't represent: • Allen's disjoint relation



[Dechter, Meiri, Pearl 91]



Modeling Visualization



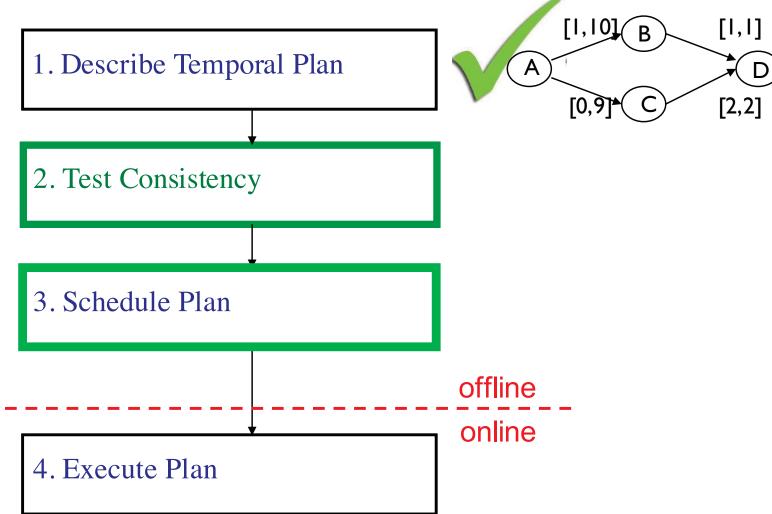
• http://bicycle.csail.mit.edu/stn/



To Execute a Temporal Plan



Schedule Offline

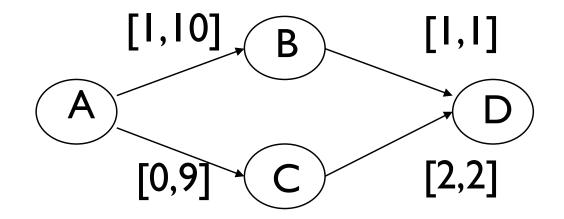




Consistency of an STN



Input: STN <X, C> where $C_j = < <X_k, X_i>, <a_j, b_j> >$



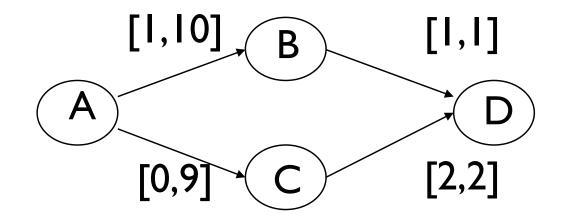
STN is **consistent** iff there exists an assignment to times X satisfying C.



Schedule of an STN



Input: STN where
$$C_j = < , >$$

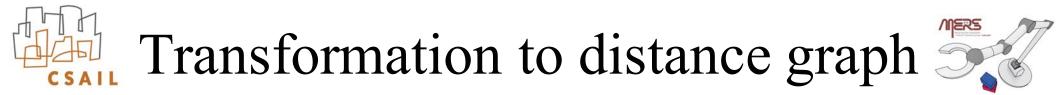


Schedule is assignment to all timepoints X consistent with constraints.





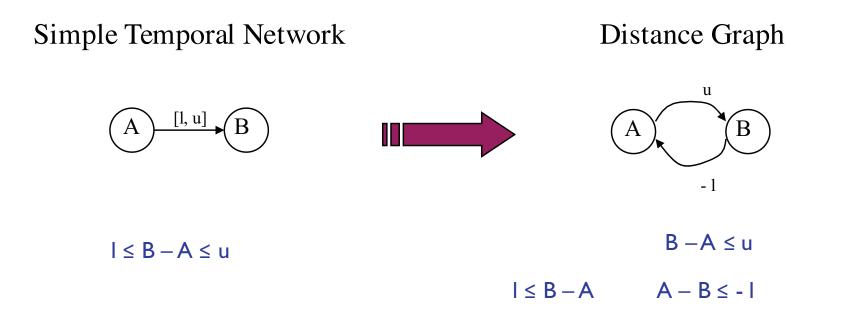
- Idea: Transform STN
 - Transform to *distance-graph*
 - Common graph algorithms (i.e., shortest path) will apply



• (Board)







- Upperbound mapped to outgoing, non-negative arc.
- Lowerbound mapped to incoming, negative arc.

[Dechter, Meiri, Pearl 91]





Initialize execution window to $[-\infty, \infty]$ for each event while unexecuted events:

 x_i = pick any unexecuted event t_i = pick any time in x_i 's execution window Propagate to all x_i 's neighbors & update their windows



Naïve (and wrong) scheduling



• (Board)

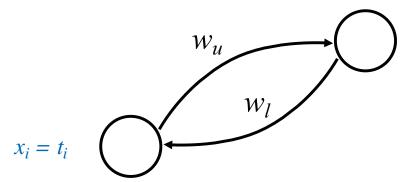




Tighten neighbor's execution windows:

- outgoing edges to neighbor: $u' = \min(u, t_i + w_u)$
- incoming edges from neighbor: $l' = \max(l, t_i w_l)$

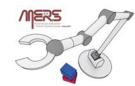


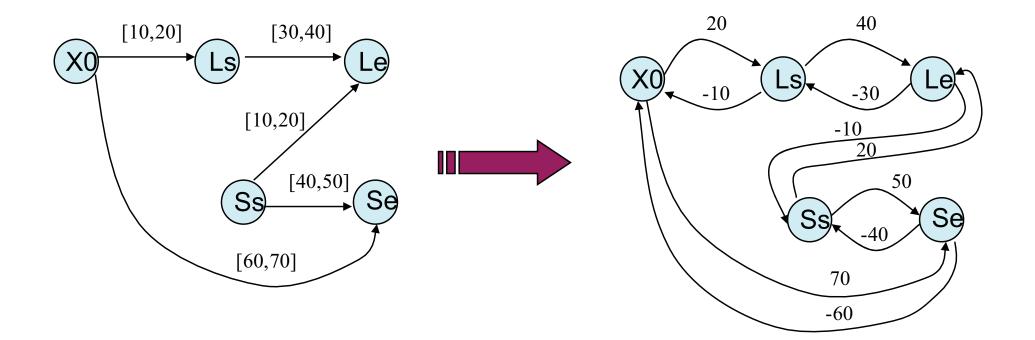




• (Board)









APSP Graph: Windows of Feasible Values



	X ₀	Ls	Le	Ss	Se	
X ₀	0	20	50	30	70	
Ls	-10	0	40	20	60	
Le	-40	-30	0	-10	30	
Ss	-20	-10	20	0	50	
Se	-60	-50	-20	40	0	

- Earliest Times APSP d-graph

	20	40	
(+ Latest Times	×0 -10	-10 -10 -10 -10 -0 -0 -0 -60	50 Se

- •Ls in [10, 20]
- •Le in [40, 50]
- •Ss in [20, 30]
- •Se in [60, 70]





- An STN or distance graph is *dispatchable* if:
 - Can be properly scheduled via local propagations to neighbors only
- Requires all implicit constraints be explicit



Checking Consistency



• (Board)

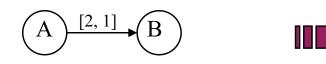


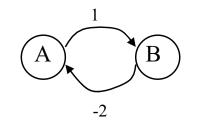
Check D-Graph Consistency



- Consistent iff D-graph has no negative cycles.
- Detect by computing shortest path from one event to all others.
 - Single Source Shortest Path (SSSP).
 - Event must reach all others.

Example of inconsistent constraint:





Simple Temporal Network

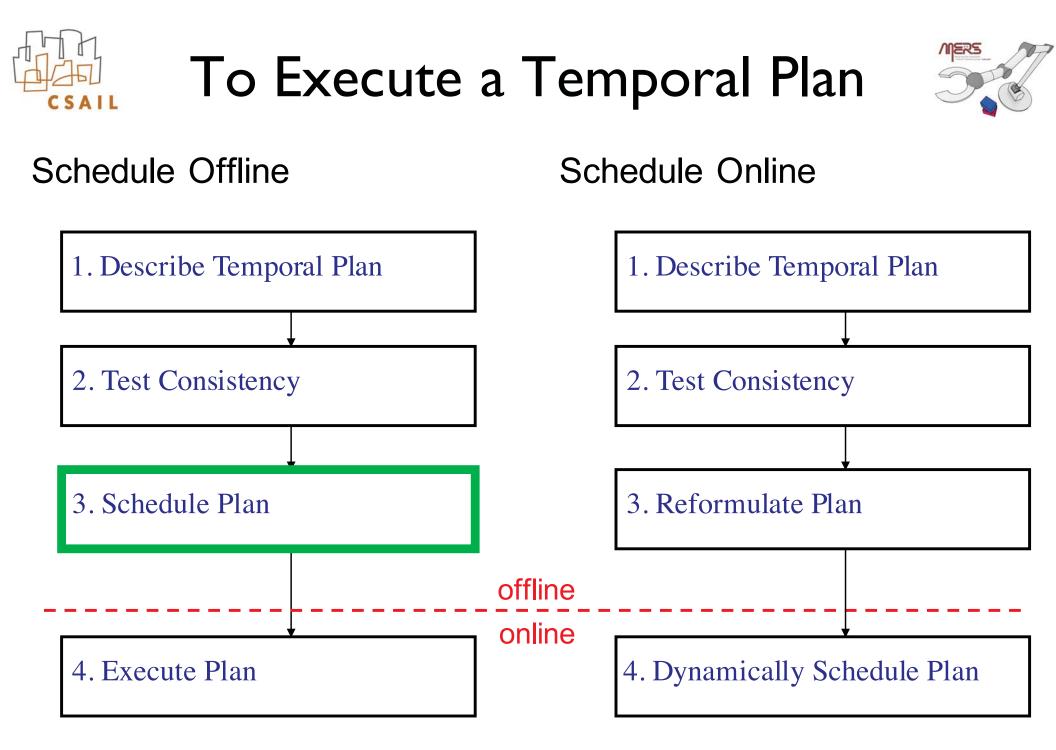
Distance Graph



Summary



- To schedule, want a simple, local-propagation algorithm
 - Requires exposing implicit constraints
- All-pairs shortest path (APSP) exposes all implicit constraints
 - Puts network in *dispatchable form*
- Negative cycle in APSP: inconsistent.







Compute dispatchable form (i.e., APSP)

Initialize execution window to $[-\infty, \infty]$ for each event while unexecuted events:

 $x_i = \text{pick}$ any unexecuted event

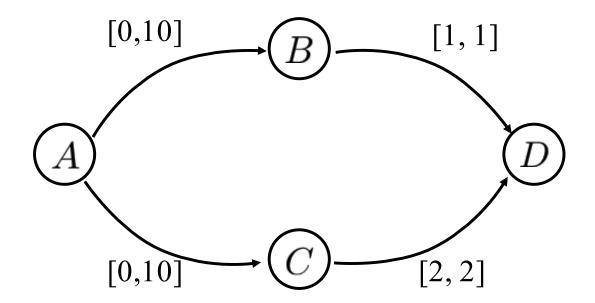
 t_i = pick any time in x_i 's execution window

Propagate to all x_i 's neighbors & update their windows



The original STN

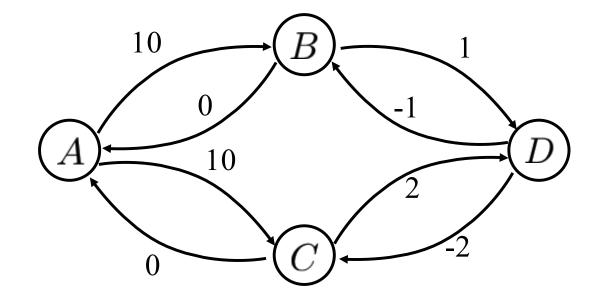






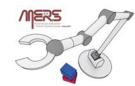
Distance graph transformation



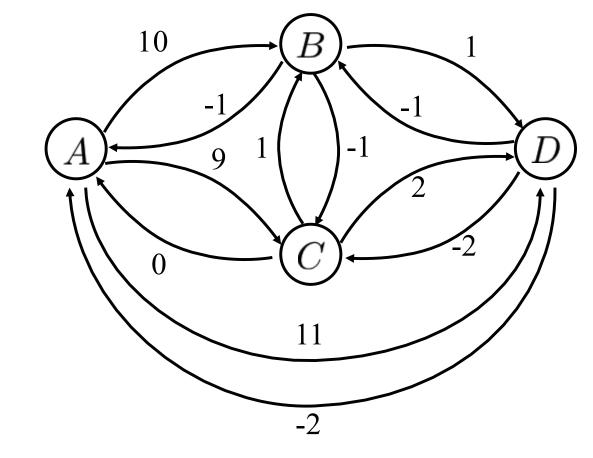




All pairs shortest path

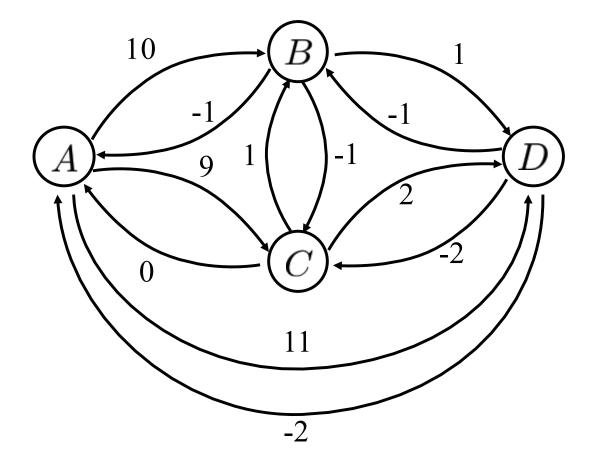


	A	В	С	D
A	0	10	9	11
В	-1	0	-1	1
С	0	1	0	2
D	-2	-1	-2	0



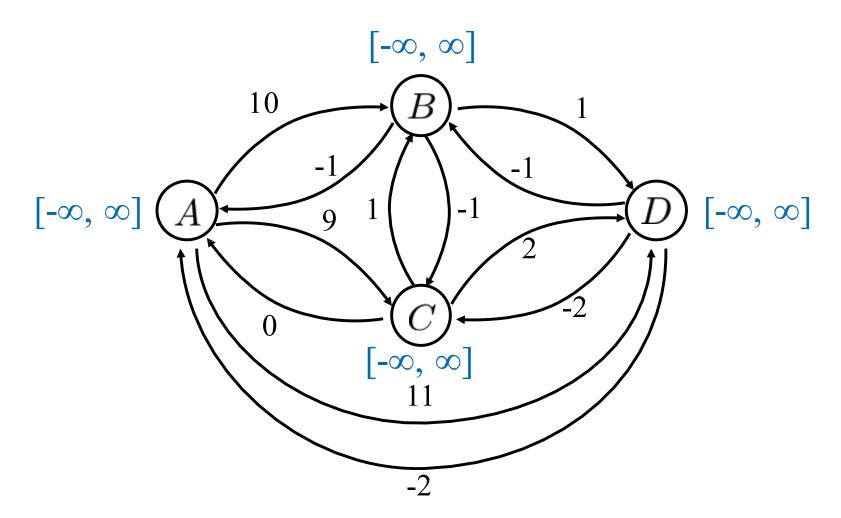








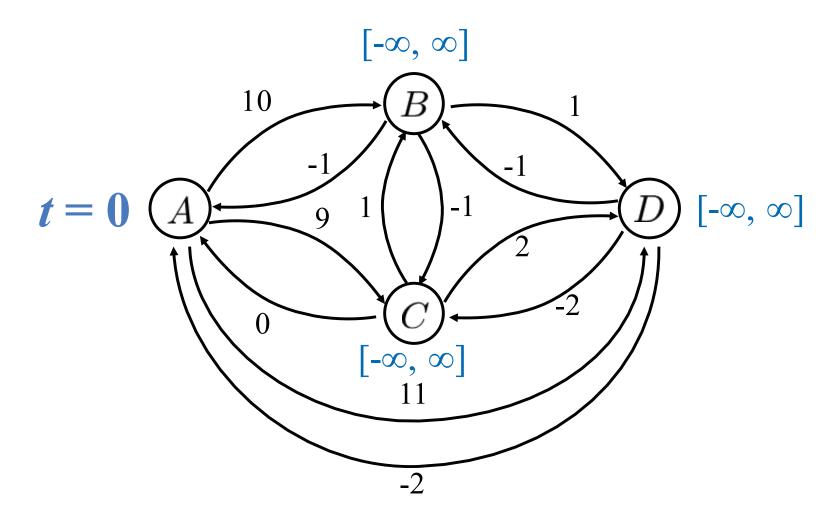




Initialize execution windows for each event in the plan





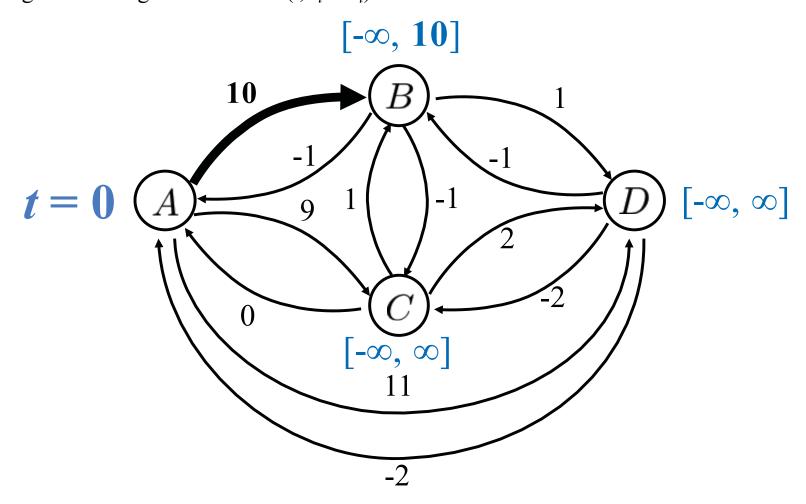


Assign the first event





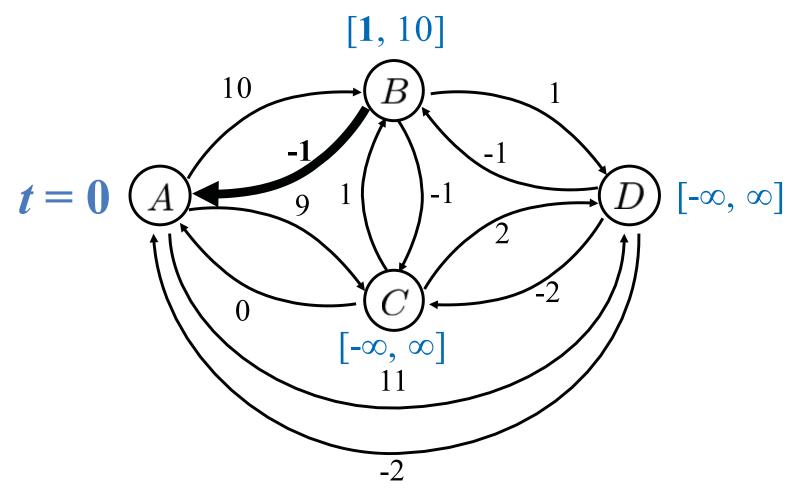
outgoing edges to neighbor: $u' = \min(u, t_i + w_u)$ incoming edges from neighbor: $l' = \max(l, t_i - w_l)$

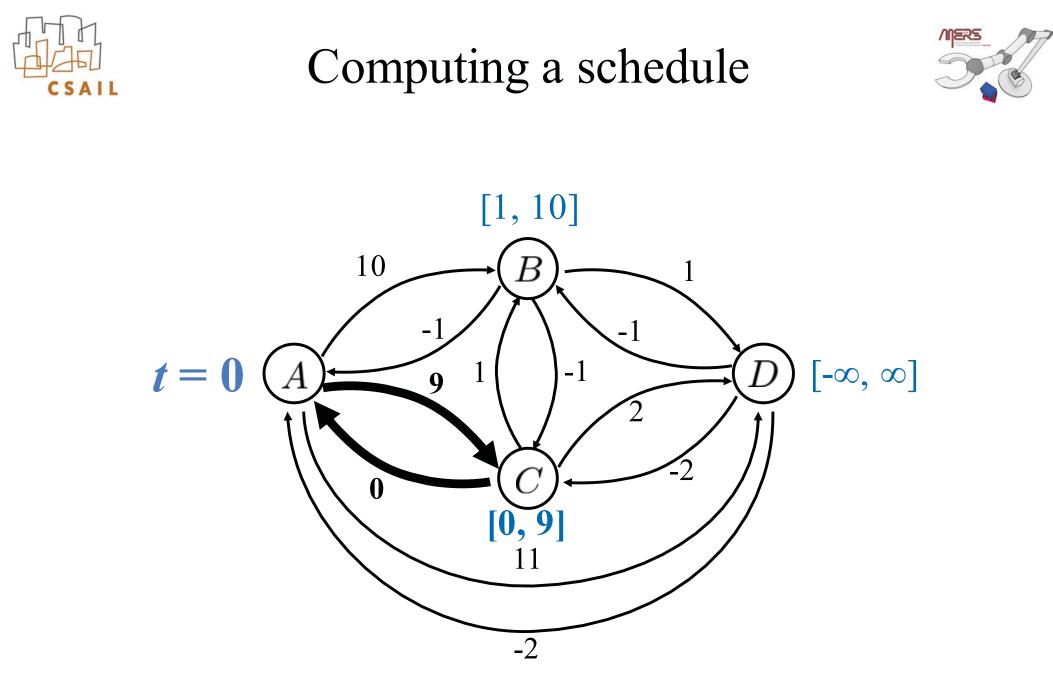


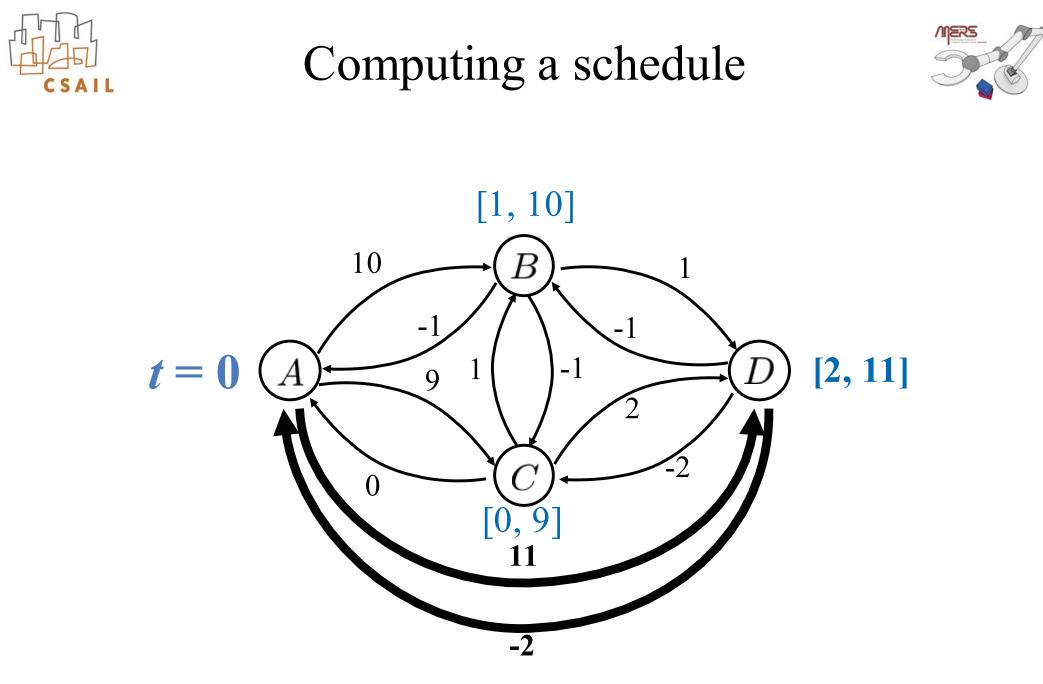




outgoing edges to neighbor: $u' = \min(u, t_i + w_u)$ incoming edges from neighbor: $l' = \max(l, t_i - w_l)$

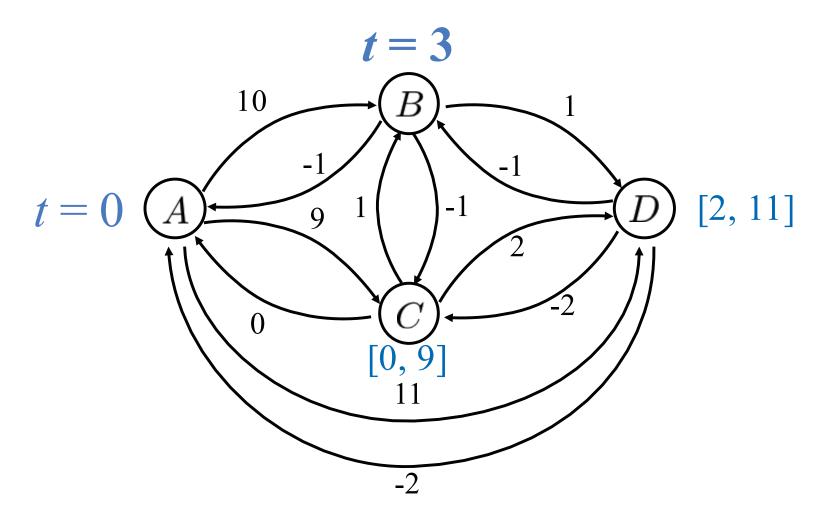








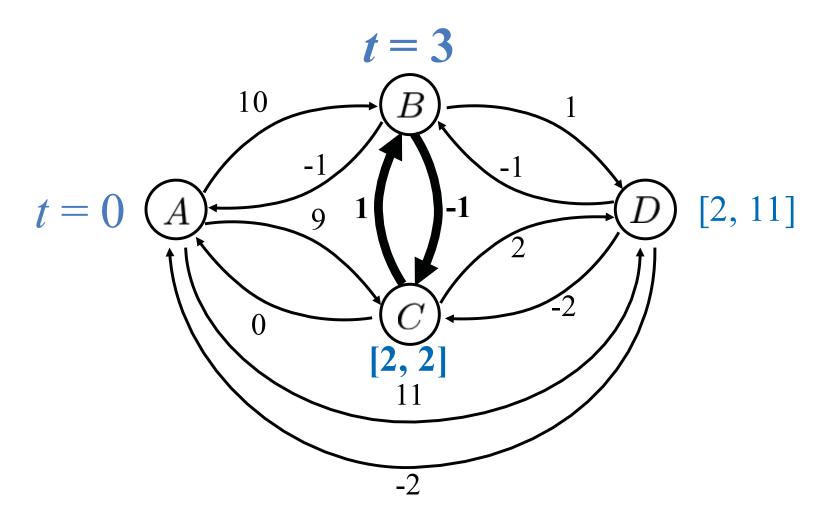




Arbitrarily pick another time point and assign it...

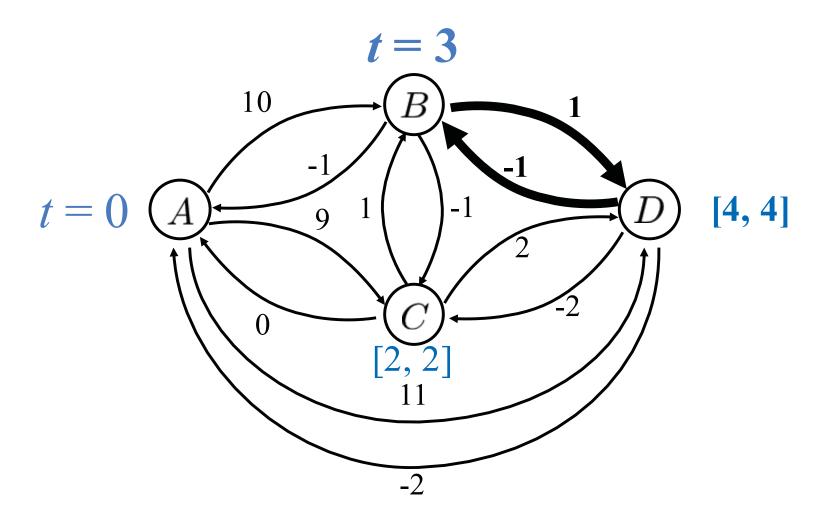






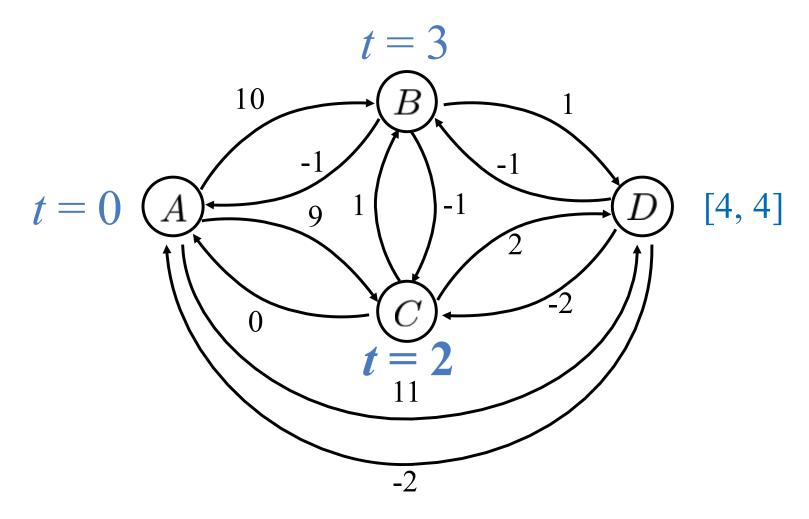








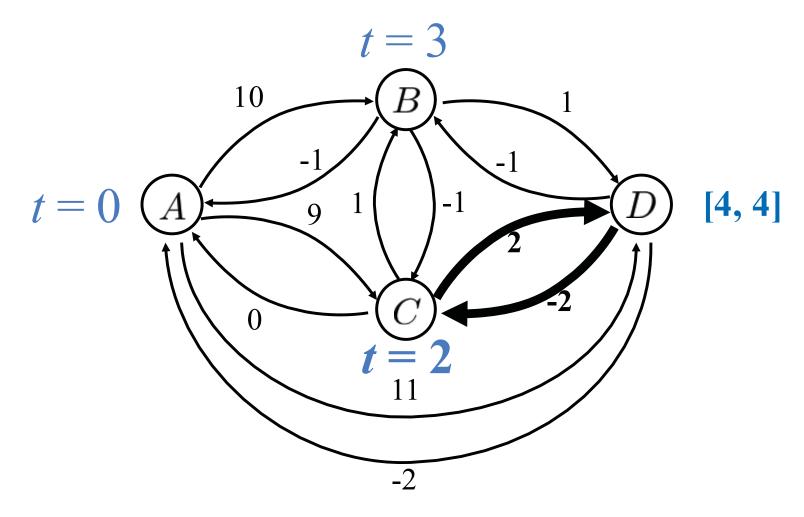




Pick another event and assign it



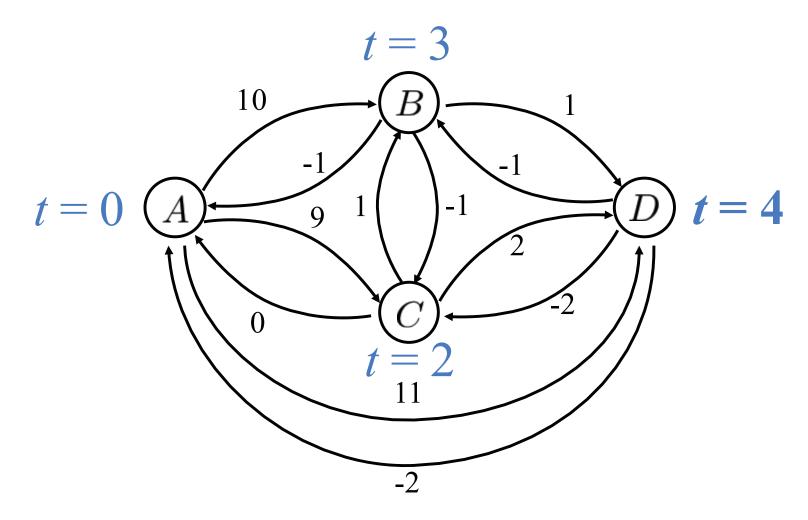




Propagate to neighbors







Assign the final event



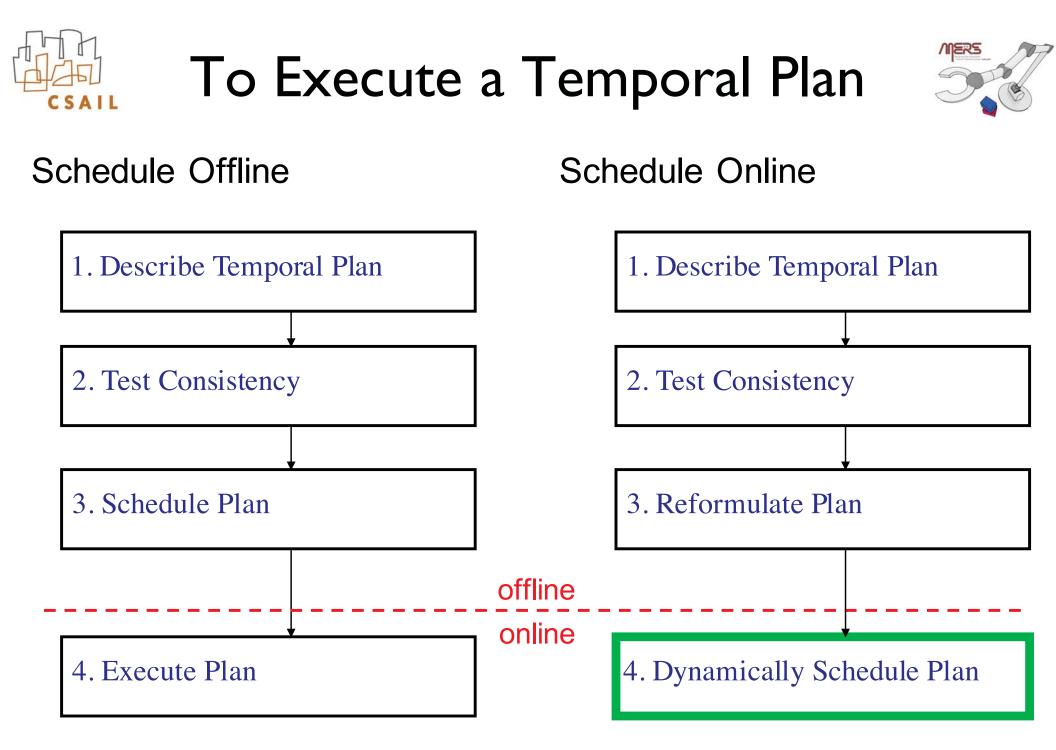
Pre-computed schedules not robust against fluctuations



• We've just computed a schedule:

 $t_A = 0, t_B = 3, t_C = 2, t_D = 4$

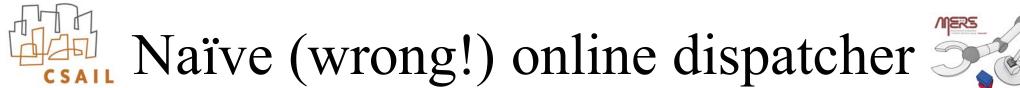
- But what if there's a disturbance?
 - i.e., what if $t_B = 3.1?$
 - i.e., what if $t_B = 4$?
 - $i.e., what if t_B = 100?$
- Pre-computed schedules not robust against fluctuations!
- **Solution:** Dispatch dynamically online.
 - Schedule events "on the fly," after observing past event times.
 - Increases robustness to many unanticipated fluctuations.
 - Flexible temporal constraints allow this!







- First, consider naive (incorrect!) approach.
- Similar to offline schedule algorithm, but now online: - Wait until current time in execution window ("active")
- (Still a problem though as we'll see shortly)



Compute dispatchable form (i.e., APSP) Initialize execution window to $[-\infty, \infty]$ for each event while unexecuted events:

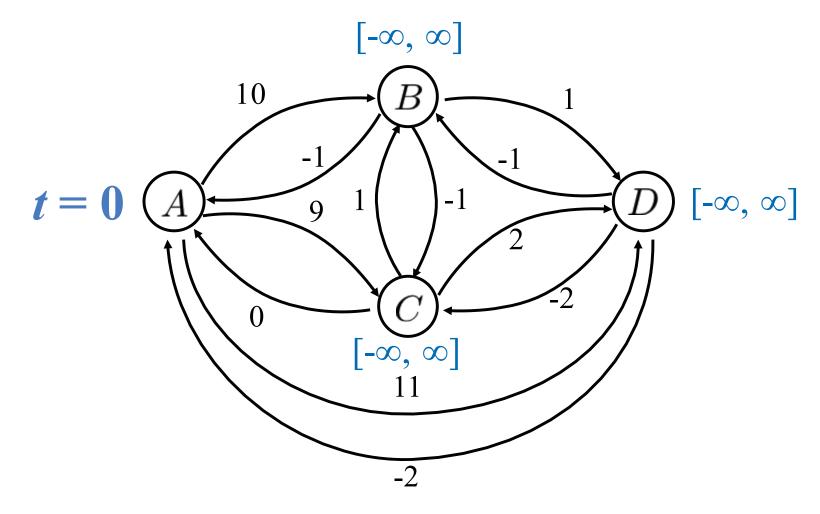
> x_i = pick any unexecuted event if current time in window $t_i = \mathbf{now}$

Propagate to all x_i 's neighbors & update their windows



Naïve (wrong!) online scheduling

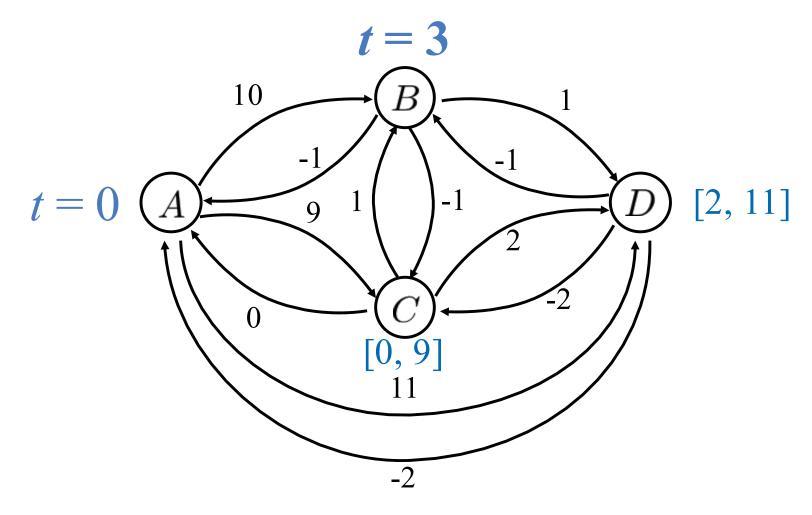




Arbitrarily picking next event...





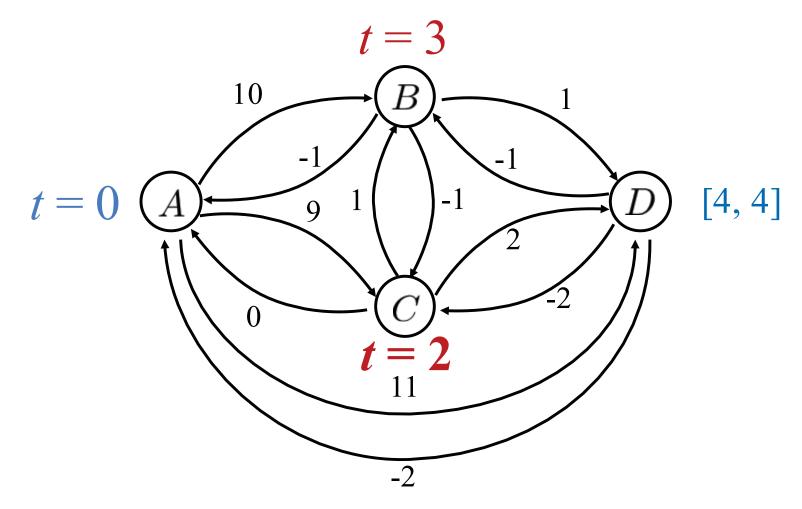


Arbitrarily picking next event...



Naïve (wrong!) online scheduling





...but wait! We just assigned a past time!





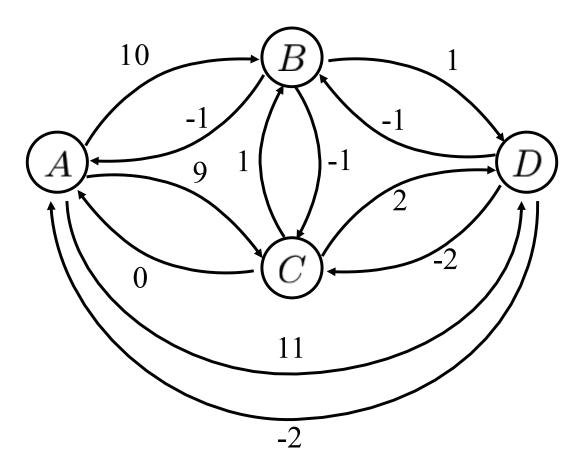
- Online: must assign monotonically increasing times
 - whereas offline algorithms may assign in any order.
- How can we constrain dispatcher to do this?
- <u>Solution</u>: determine "enablement conditions" by analyzing negative edges.

– Allows us to infer if some edges must precede other edges





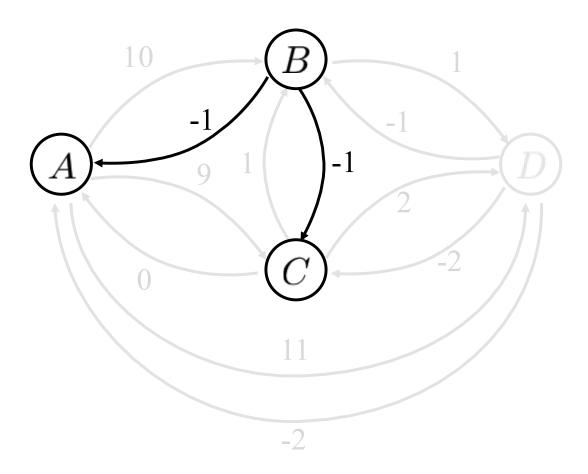
• Negative edges from APSP dictate ordering constraints







• Negative edges from APSP dictate ordering constraints



B must occur after both A and C!





• An event is **enabled** if all its neighbors over negative edges have already been dispatched.

– All "predecessors" have been dispatched.

- Modify online dispatching algorithm to only dispatch events if they are enabled.
- An event is *active* is the current time is within that event's execution window





Compute dispatchable form (i.e., APSP) Initialize execution window to $[-\infty, \infty]$ for each event $E \leftarrow \{\text{events with no predecessors}\} \# \text{ set of enabled events}$ $S \leftarrow \{\} \# \text{ set of executed events}$

while unexecuted events:

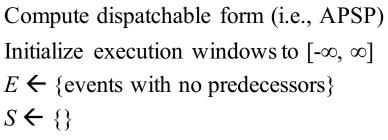
Wait until some event x_i in E is active

 $t_i = \text{now } \# \text{ dispatch } x_i \text{ now at } t_i$

Propagate to all x_i 's neighbors & update their windows Add x_i to S

Add to *E* any now-enabled events



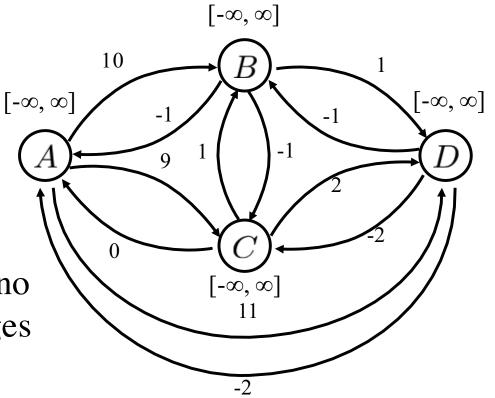


while unexecuted events:

Wait until some event x_i in E is active $t_i = now$ Propagate to x_i 's neighbors Add x_i to SAdd to E any now-enabled events

A, C initially in E – have no negative, outgoing edges

 $E = \{A, C\}$ $S = \{\}$







Compute dispatchable form (i.e., APSP) Initialize execution windows to $[-\infty, \infty]$ $E \leftarrow \{\text{events with no predecessors}\}$ $S \leftarrow \{\}$

while unexecuted events:

Wait until some event x_i in E is active

 $t_i = now$

Propagate to x_i 's neighbors

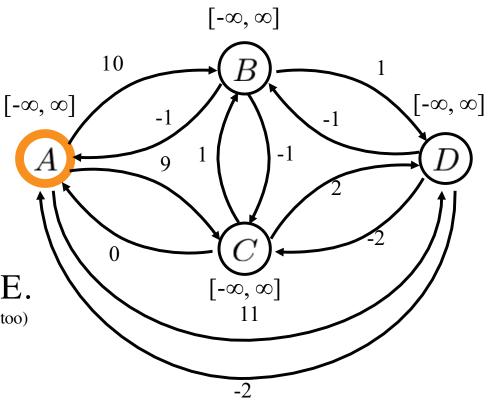
Add x_i to S

Add to E any now-enabled events

A is enabled and in E.

(could have chosen C too)

 $E = \{A, C\}$ $S = \{\}$







Compute dispatchable form (i.e., APSP) Initialize execution windows to $[-\infty, \infty]$ $E \leftarrow \{\text{events with no predecessors}\}$ $S \leftarrow \{\}$

while unexecuted events:

Wait until some event x_i in E is active

 $t_i = now$

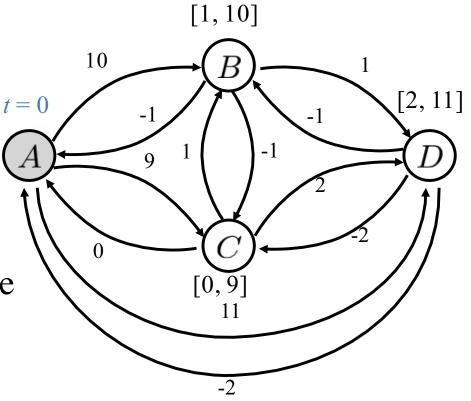
Propagate to x_i 's neighbors

Add x_i to S

Add to E any now-enabled events

Dispatch A and propagate

 $E = \{C\}$ $S = \{A\}$







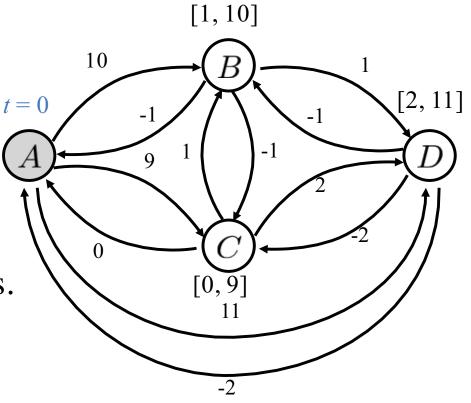
Compute dispatchable form (i.e., APSP) Initialize execution windows to $[-\infty, \infty]$ $E \leftarrow \{\text{events with no predecessors}\}$ $S \leftarrow \{\}$

while unexecuted events:

Wait until some event x_i in E is active $t_i = now$ Propagate to x_i 's neighbors Add x_i to SAdd to E any now-enabled events

B, D not enabled! But C still is.

 $E = \{C\}$ $S = \{A\}$







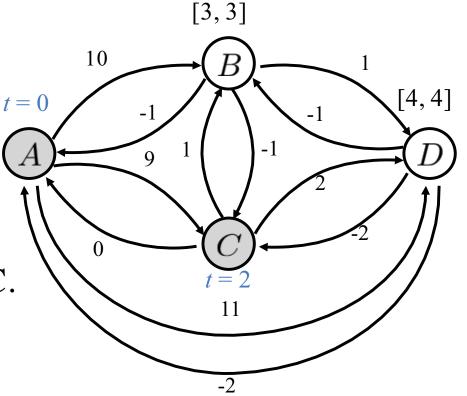
Compute dispatchable form (i.e., APSP) Initialize execution windows to $[-\infty, \infty]$ $E \leftarrow \{\text{events with no predecessors}\}$ $S \leftarrow \{\}$ while unexecuted events: Wait until some event x_i in *E* is active

 $t_i = now$ Propagate to x_i 's neighbors Add x_i to S

Add to E any now-enabled events

Dispatch & propagate C.

 $E = \{\}$ $S = \{A, C\}$







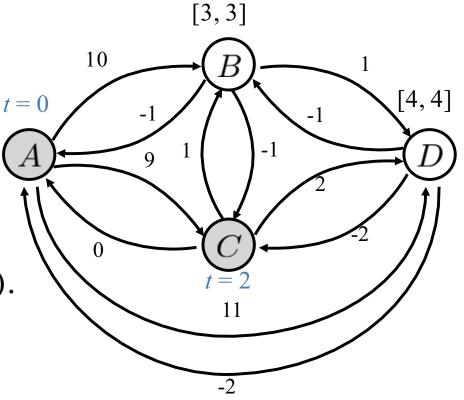
Compute dispatchable form (i.e., APSP) Initialize execution windows to $[-\infty, \infty]$ $E \leftarrow \{\text{events with no predecessors}\}$ $S \leftarrow \{\}$

while unexecuted events:

Wait until some event x_i in E is active $t_i = now$ Propagate to x_i 's neighbors Add x_i to SAdd to E any now-enabled events

B is now enabled (but still not D).

 $E = \{B\}$ $S = \{A, C\}$







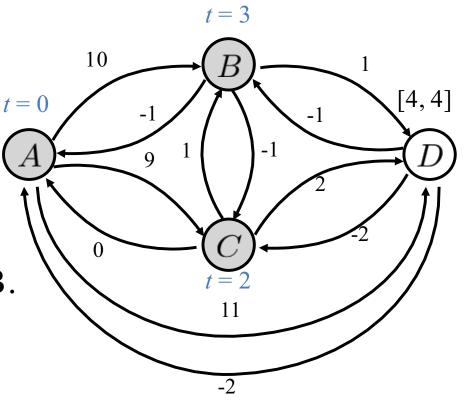
Compute dispatchable form (i.e., APSP) Initialize execution windows to $[-\infty, \infty]$ $E \leftarrow \{\text{events with no predecessors}\}$ $S \leftarrow \{\}$

while unexecuted events:

Wait until some event x_i in E is active $t_i = now$ Propagate to x_i 's neighbors Add x_i to SAdd to E any now-enabled events

Dispatch & propagate B.

 $E = \{\}$ $S = \{A, C, B\}$







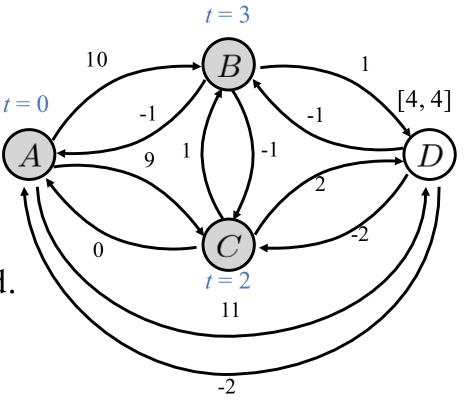
Compute dispatchable form (i.e., APSP) Initialize execution windows to $[-\infty, \infty]$ $E \leftarrow \{\text{events with no predecessors}\}$ $S \leftarrow \{\}$

while unexecuted events:

Wait until some event x_i in E is active $t_i = now$ Propagate to x_i 's neighbors Add x_i to SAdd to E any now-enabled events

D is finally enabled.

 $E = \{\mathbf{D}\}$ $S = \{\mathbf{A}, \mathbf{C}, \mathbf{B}\}$







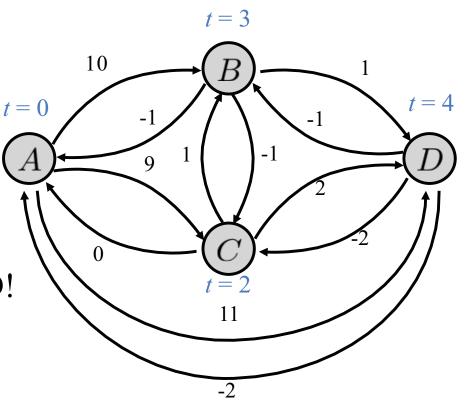
Compute dispatchable form (i.e., APSP) Initialize execution windows to $[-\infty, \infty]$ $E \leftarrow \{\text{events with no predecessors}\}$ $S \leftarrow \{\}$

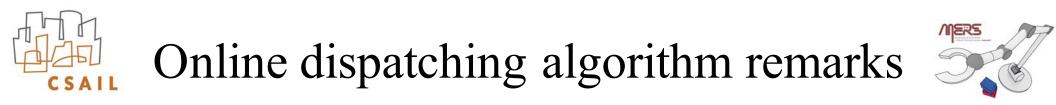
while unexecuted events:

Wait until some event x_i in E is active $t_i = now$ Propagate to x_i 's neighbors Add x_i to SAdd to E any now-enabled events

Finish up by dispatching D!

 $E = \{\}$ $S = \{A, C, B, D\}$





- By considering predecessors, we guarantee that events assigned monotonically increasing times online.
- Capable of responding to fluctuations that do not affect overall temporal feasibility.
- (Note: must be run on an dispatchable / APSP graph!)





• Consider an STN with *n* edges.

• How many edges in APSP distance graph?





• Consider an STN with *n* edges.

• How many edges in APSP distance graph? n^2 .





- Consider an STN with *n* edges.
- How many edges in APSP distance graph? n^2 .
- How many neighbors to propagate to each step?





- Consider an STN with *n* edges.
- How many edges in APSP distance graph? n^2 .
- How many neighbors to propagate to each step? *n*.





- Consider an STN with *n* edges.
- How many edges in APSP distance graph? n^2 .
- How many neighbors to propagate to each step? *n*.
- Large STNs: propagation slow. Want to reduce this.

```
Compute dispatchable form (i.e., APSP)

Initialize execution windows to [-\infty, \infty]

E \leftarrow \{\text{events with no predecessors}\}

S \leftarrow \{\}

while unexecuted events:

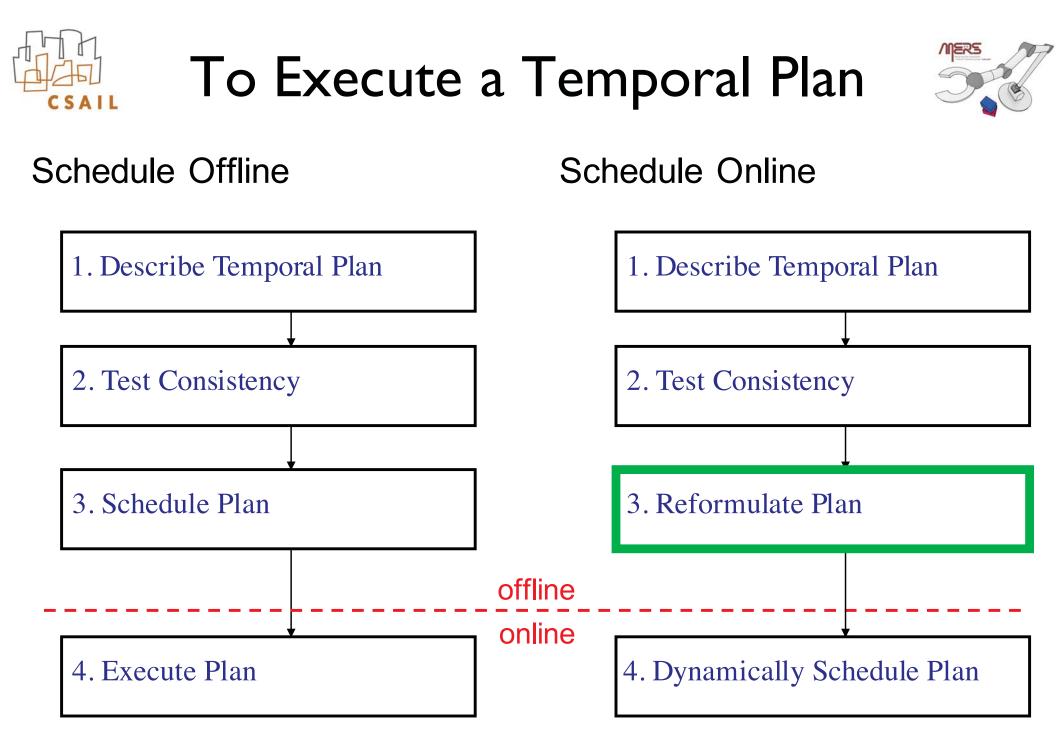
Wait until some event x_i in E is active

t_i = \text{now}

Propagate to x_i's neighbors

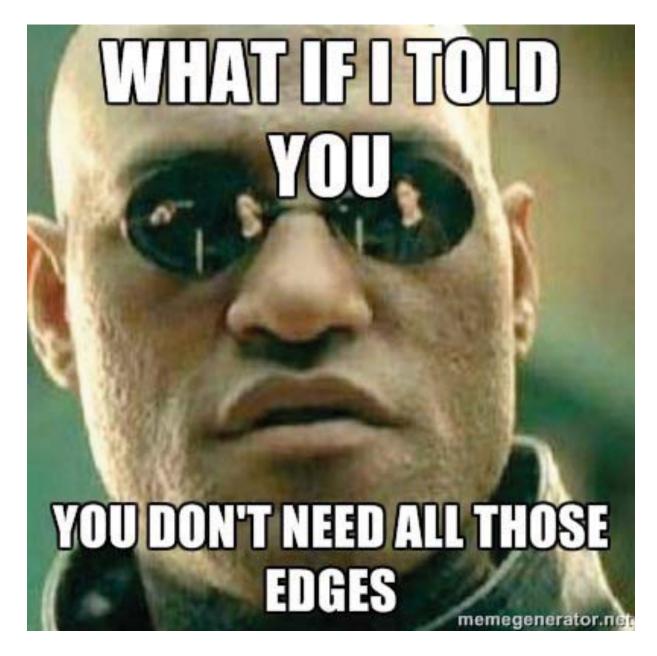
Add x_i to S

Add to E any now-enabled events
```



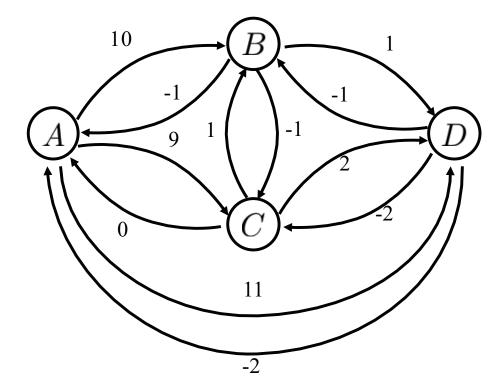






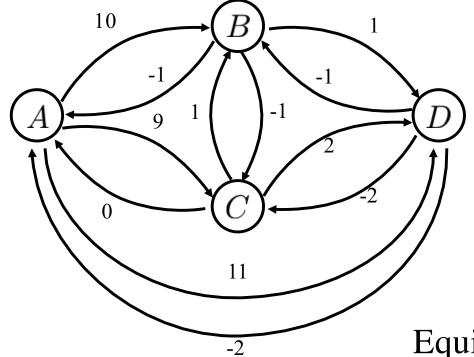


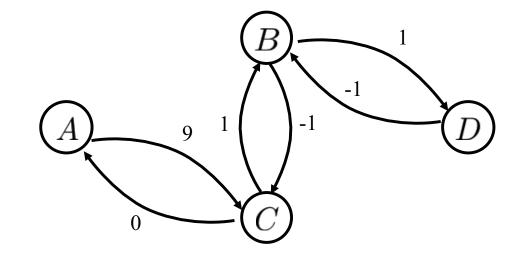










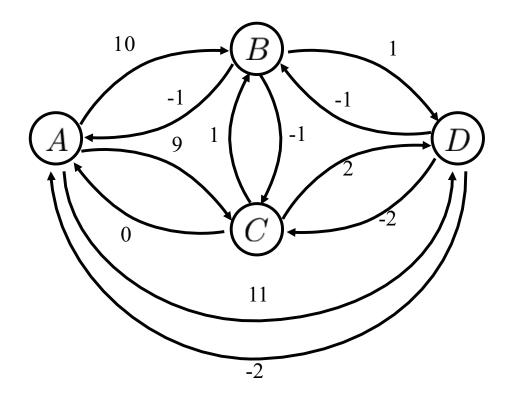


Equivalent minimal dispatchable network





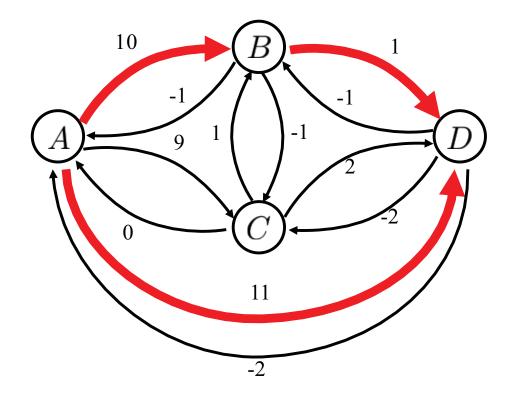
Let's consider a specific triangle of edges.







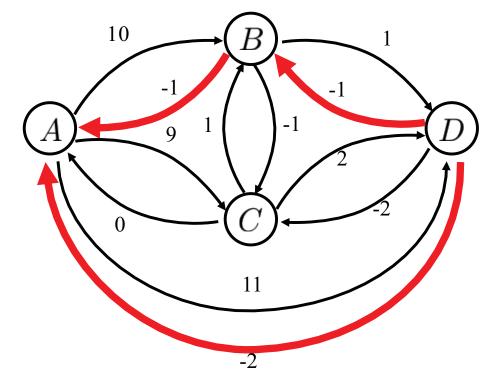
Let's consider a specific triangle of edges. Do we really need the bottom edge?

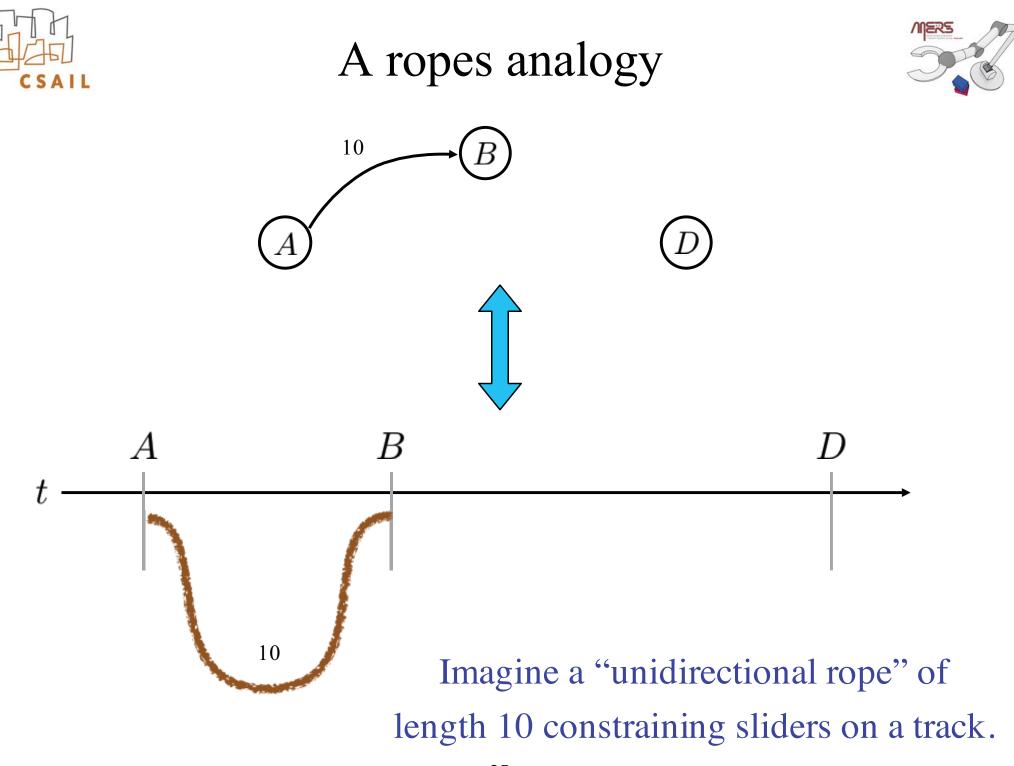


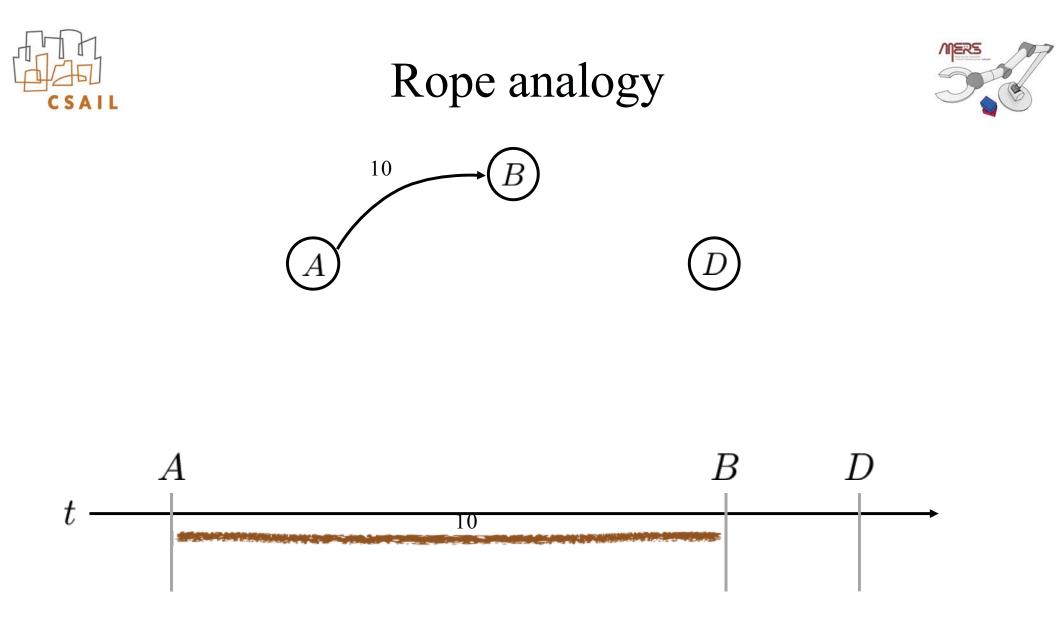




Let's consider a different triangle of edges. Do we really need the bottom edge?

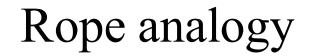




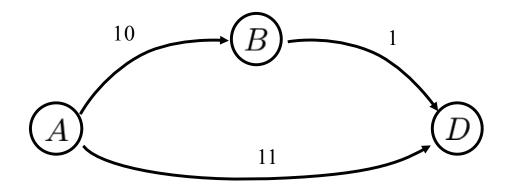


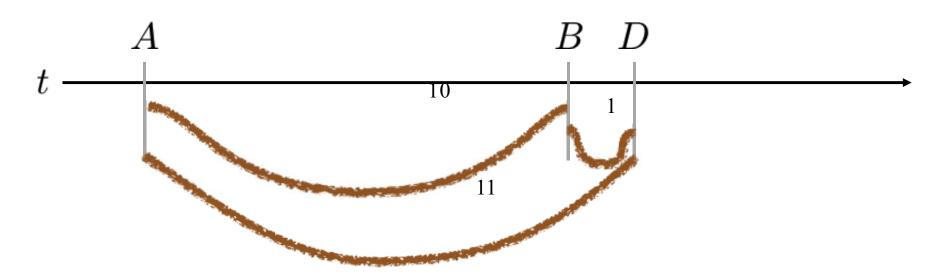
Imagine a "unidirectional rope" of length 10 constraining sliders on a track.





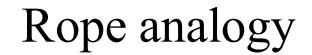




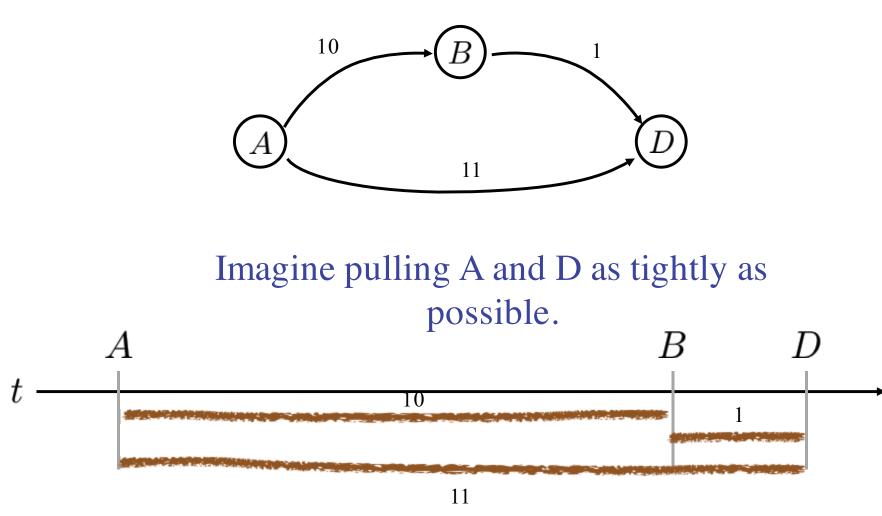


Now add in ropes for other constraints

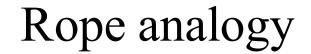




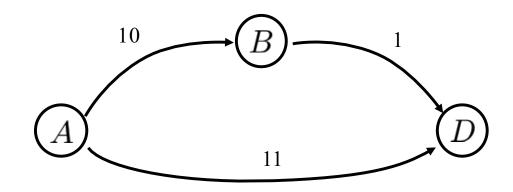




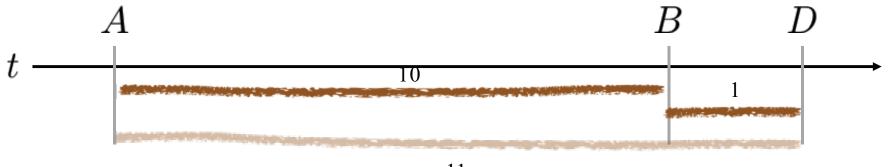








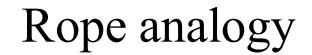
Can we remove rope AD without changing behavior?



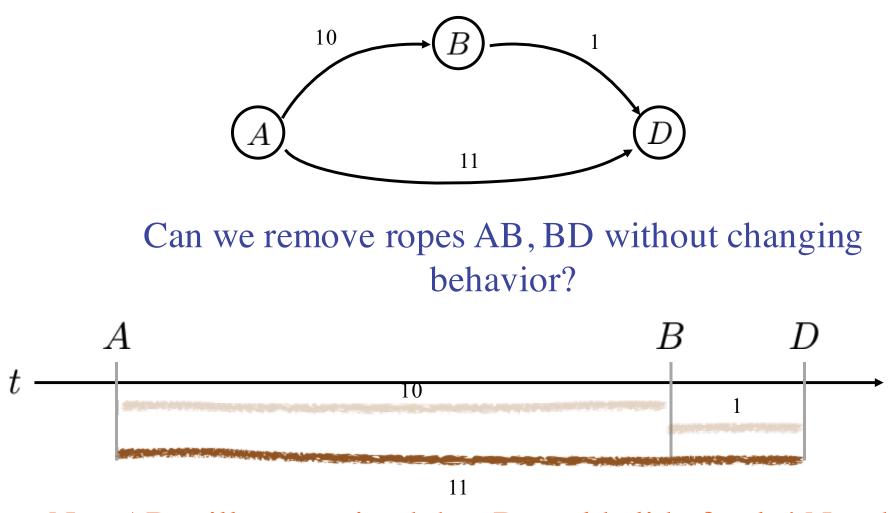
11

Yes! Same possible positions for A, B, D.







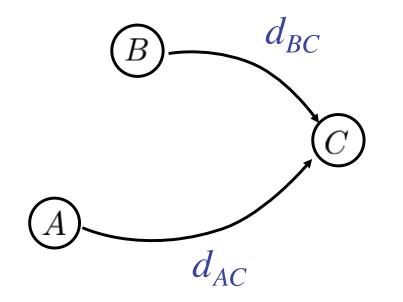


No. AD still constrained, but B could slide freely! Not the same behavior. Collectively, AB and BD entail AD (but AD does not entail both AB and AD).



Upper dominating edges - detection from APSP





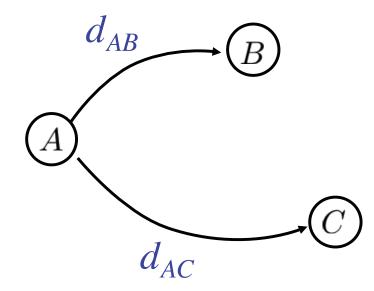
If d_{AC} , $d_{BC} \ge 0$ and $d_{AB} + d_{BC} = d_{AC}$ then BC *dominates* AC

(Proof omitted - based on triangle rule property of APSP. Please see notes / reading for more info)



Lower dominating edges - detection from APSP



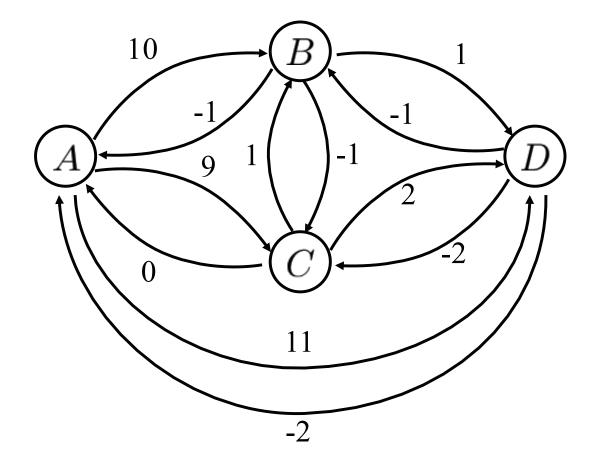


If d_{AB} , $d_{AC} < 0$ and $d_{AB} + d_{BC} = d_{AC}$ then AB *dominates* AC

(Proof omitted - based on triangle rule property of APSP. Please see notes / reading for more info)

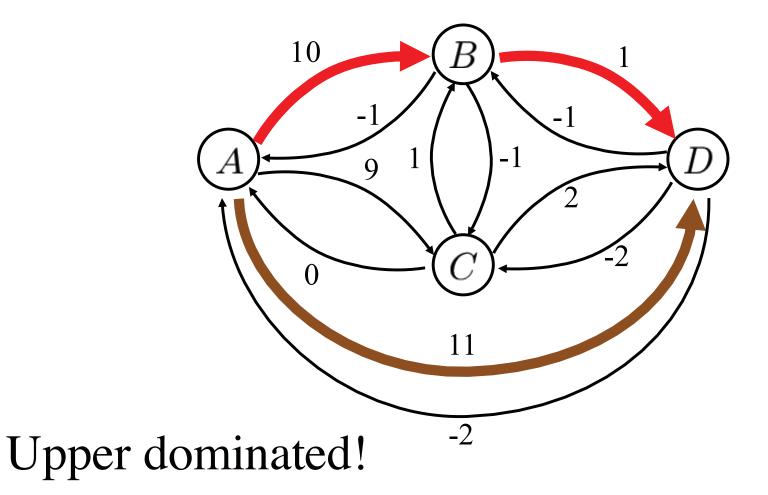






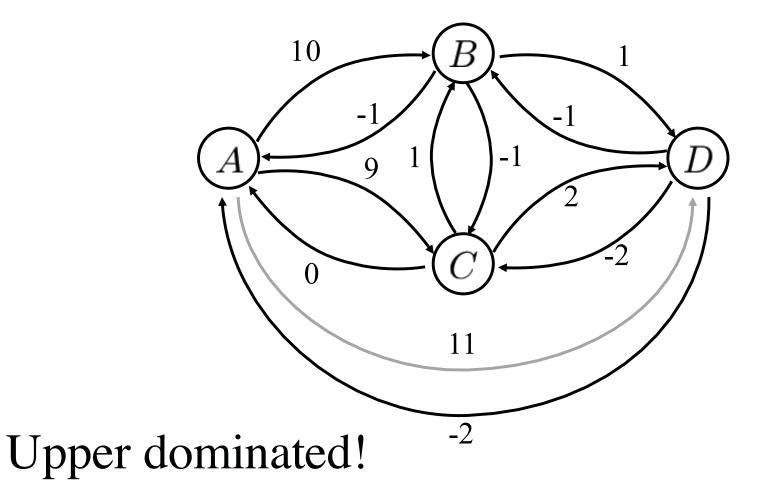








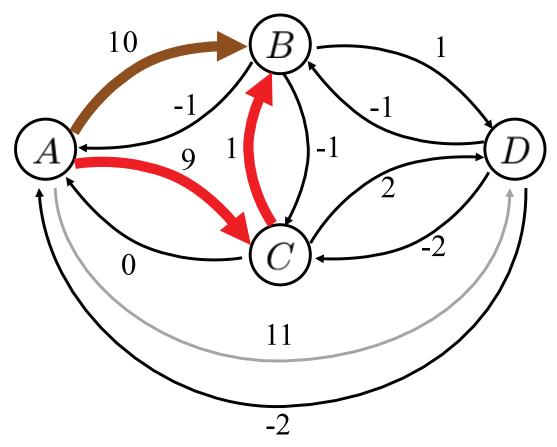








Upper dominated!

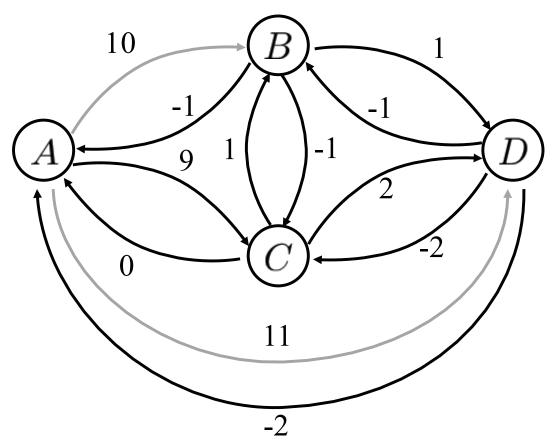




Dominance example

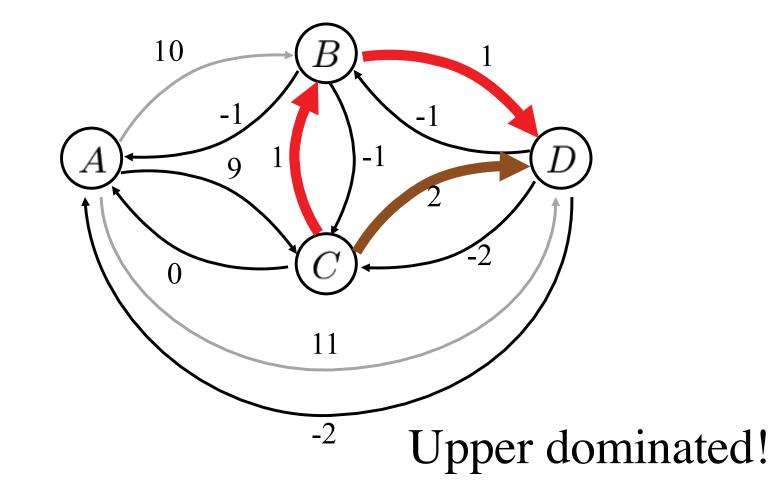


Upper dominated!



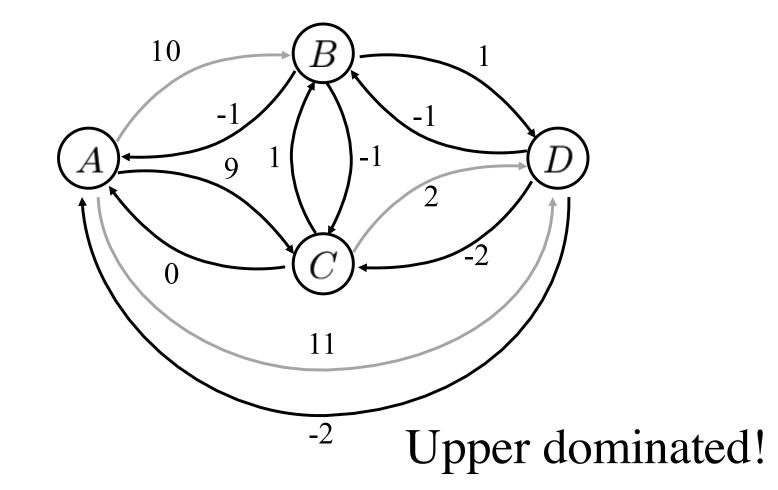










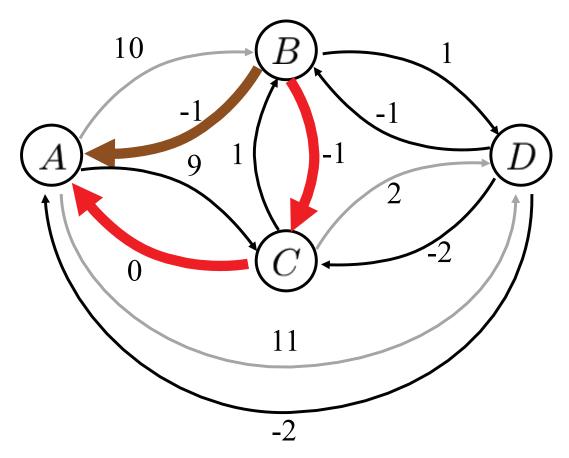








Lower dominated!

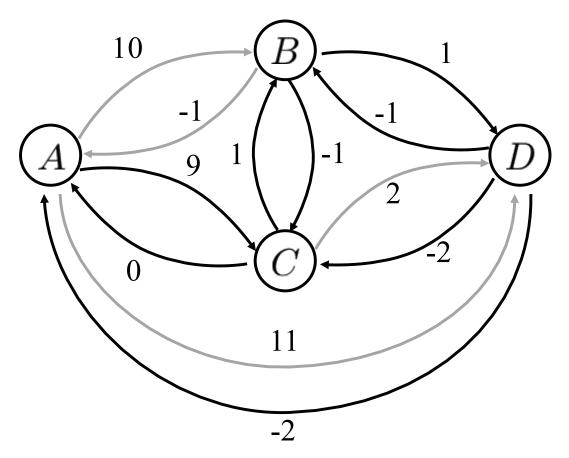






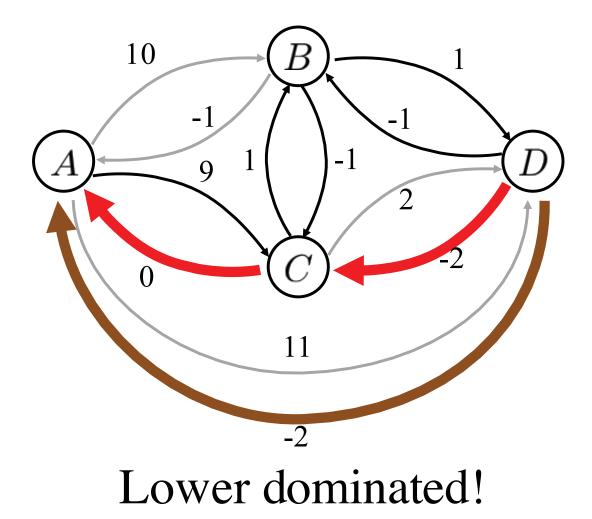


Lower dominated!



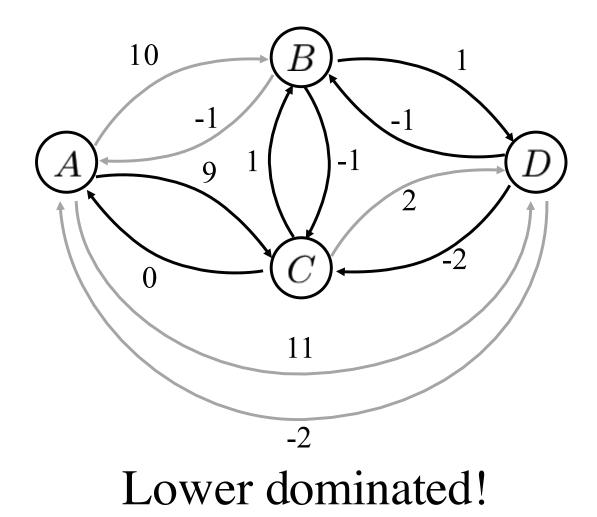






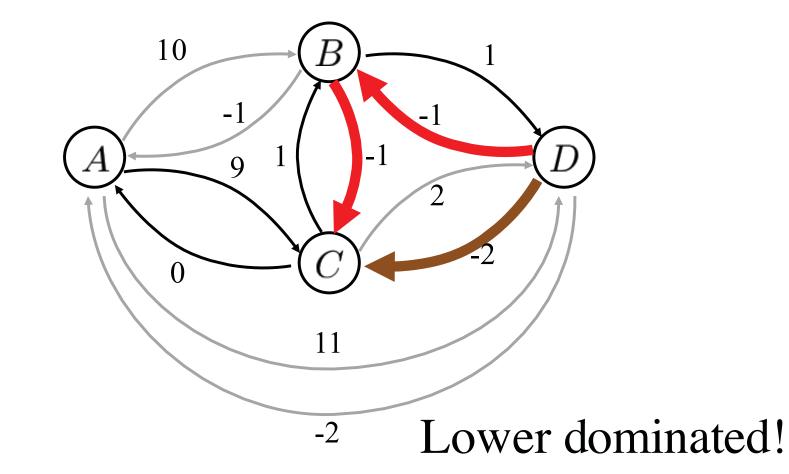






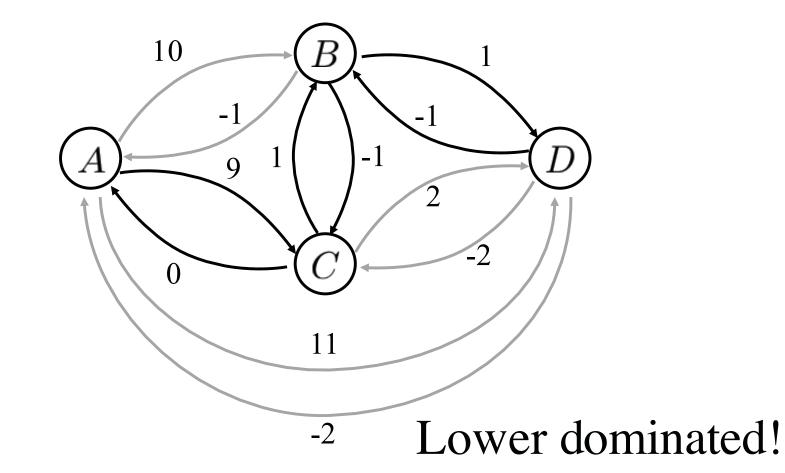






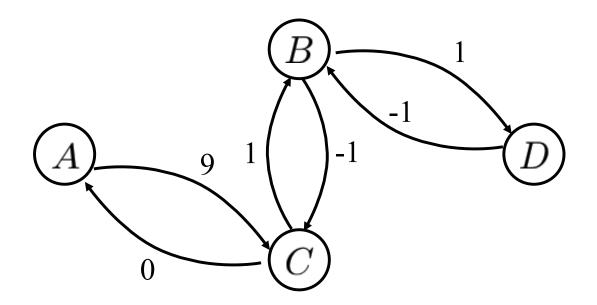






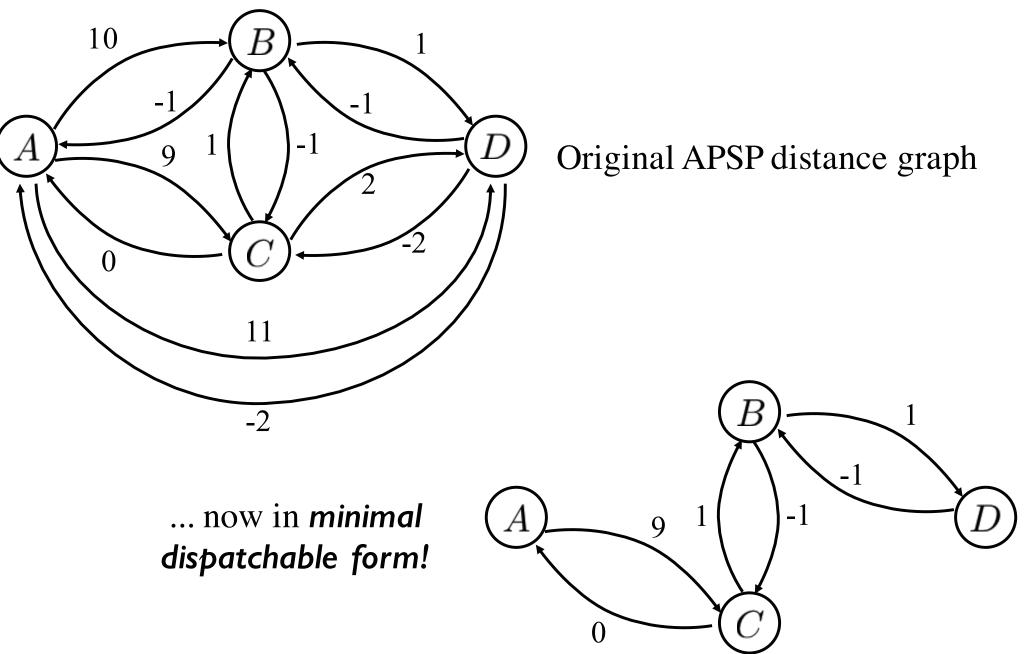






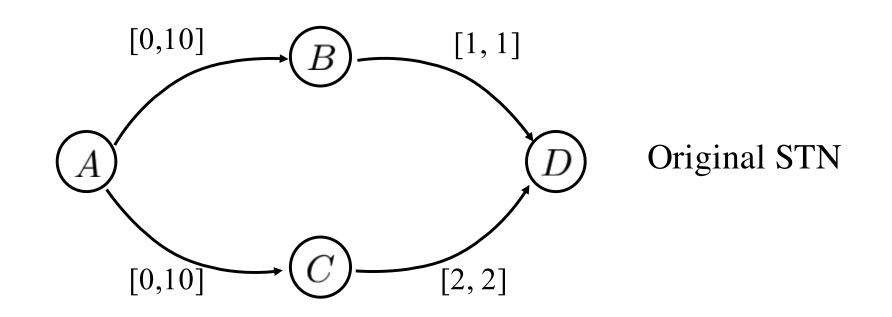


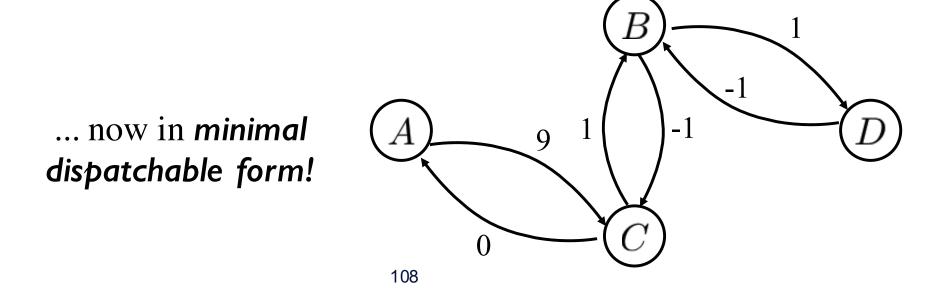


















Input: A dispatchable APSP-graph G

Output: A minimal dispatchable graph

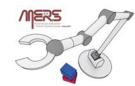
- 1 for each pair of intersecting edges in G
- 2 if both dominate each other
- 3 if neither is marked
- 4 arbitrarily mark one for elimination
- 5 end if
- 6 else if one dominates the other
- 7 mark dominated edge for elimination
- 8 end if
- 9 end for

10 remove all marked edges from graph

11 return G



Avoiding Intermediate Graph Explosion



- Problem:
 - All pairs shortest path table computation consumed $O(n^2)$ space
 - Only used as an intermediate not needed after minimal dispatchable graph obtained.
- Solution:
 - Interleave process of APSP construction with edge elimination.
 - Never have to build whole APSP graph.

[Tsarmardinos 1998]







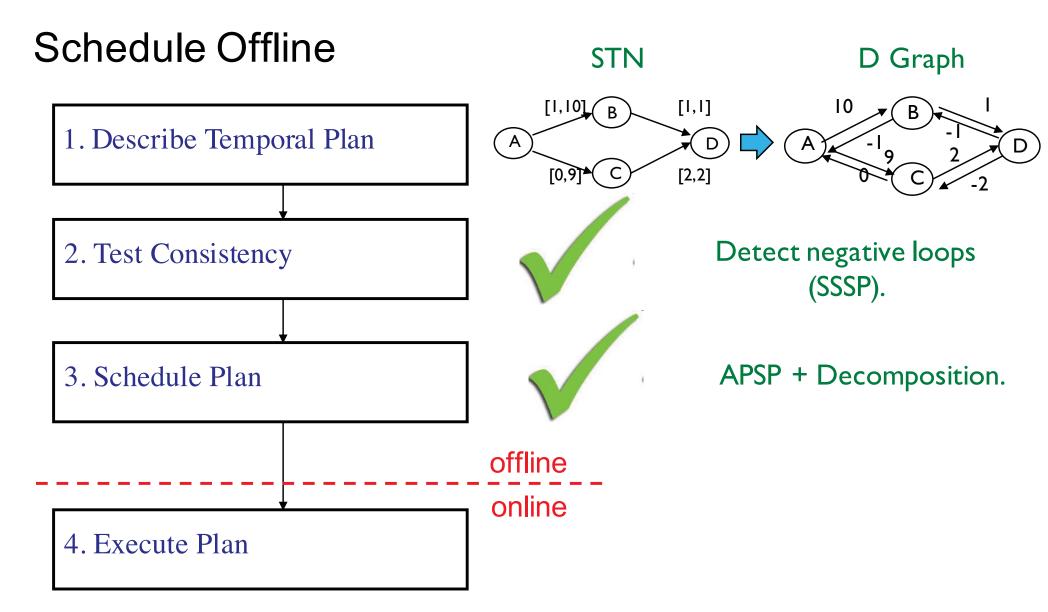
• Recap

- To schedule online, times must monotonically increase use enablement conditions
- Running online allows greater flexibility to fluctuations
- However, propagation costs can be large for large graphs
- Can reduce edges by using domination to make graph smaller



To Execute a Temporal Plan





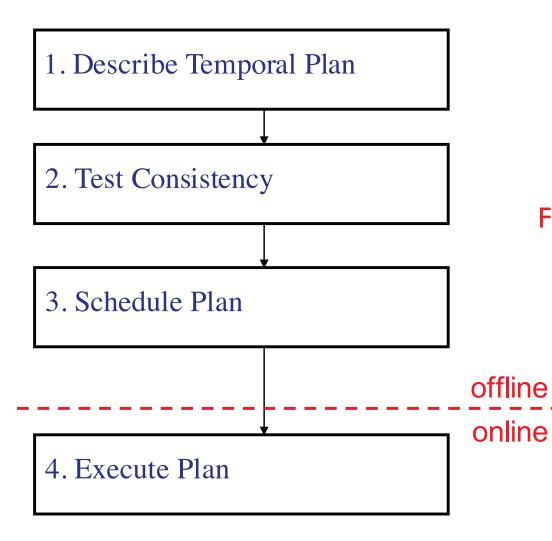
[Dechter, Meiri, Pearl 91]



To Execute a Temporal Plan



Schedule Offline



Problem: delays and fluctuations in task duration can cause plan failure.

Observation: temporal constraints leave room to adapt.

Flexible Execution adapts through dynamic scheduling:

Assign time to event when executed.

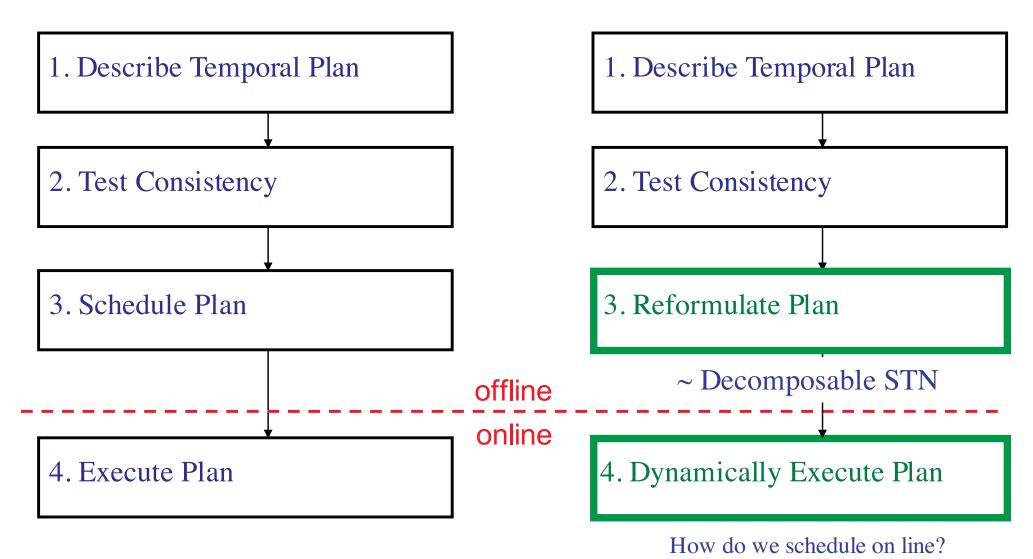
- Guarantee that all constraints will be satisfied.
- Schedule with low latency through pre-compilation.





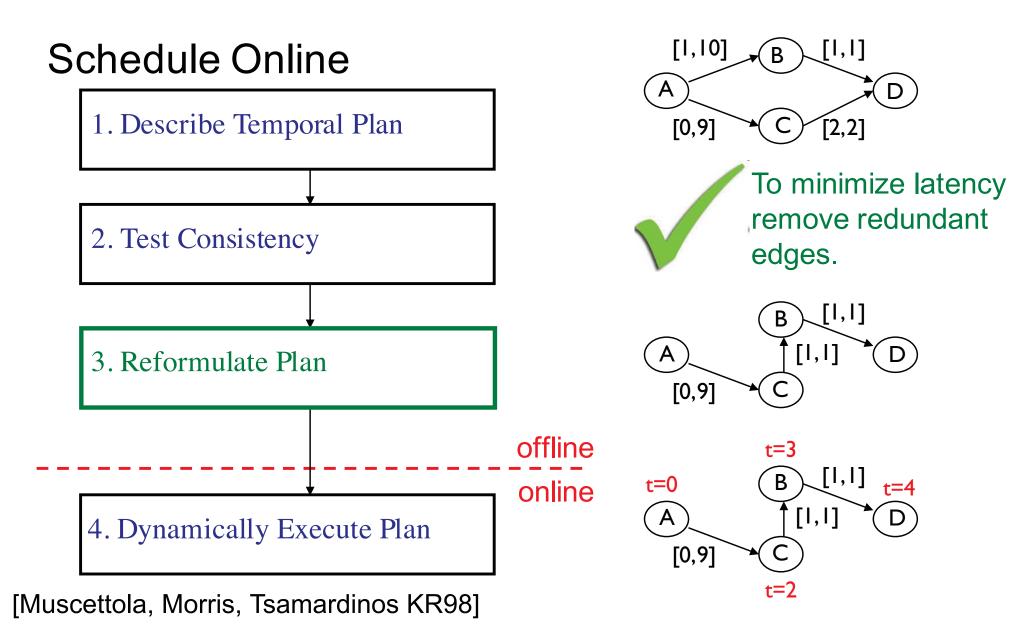
Schedule Offline

Schedule Online





Outline: To Execute a Temporal Plan



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