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### 16.36 Communication Systems Engineering

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## MASSACHUSETTS INSTITUTE OF TECHNOLOGY <br> Department of Aeronautics and Astronautics

16.36: Comm. Sys. Engineering

Problem Set No. 10
Problem 1: Suppose that X and Y are independent Poisson random variables with rate $\lambda_{1}$ and $\lambda_{2}$ respectively.
A) Show that $E[X]=\lambda_{1}$
B) Show that $E\left[X^{2}\right]=\lambda_{1}+\left(\lambda_{1}\right)^{2}$
C) Show that $\mathrm{Z}=\mathrm{X}+\mathrm{Y}$ is Poisson of rate $\lambda_{1}+\lambda_{2}$

Problem 2: Customers arrive at a movie theater at a rate of 10 per minute. They wait in line for tickets for an average of 10 minutes.
A) What is the average number of customers waiting in line?
B) After getting a movie ticket, each customer gets on line for snacks with probability 0.5 or goes straight into the theater with probability 0.5 . Each customer waits on line for snacks for an average of 5 minutes. What is the average number of people waiting in line for snacks?

Problem 3: Packets arrive to a link at an average rate of 20 per second. The transmission rate of the link is 40,000 bits per second and the average packet length is 1000 bits. Assume that packet lengths are Exponentially distributed, and that arrivals are Poisson.
A) What is the transmission rate in packets per second?
B) What is the average queuing delay?
C) What is the average number of packets in the buffer?
D) What is the probability that the system is empty?
E) Repeat parts B and C assuming that the packets are all the same length.
F) Repeat parts B and C assuming that $1 / 2$ the packets are 500 bits and $1 / 2$ are 1500 bits.

Problem 4: A group of N stations share a 1 Mbps pure (unslotted) aloha channel. Each station has one (NEW) packet arriving every 10 seconds and packets are 1000 bits long. What is the maximum value of N that the channel can accommodate?

Problem 5: A slotted aloha system has packets (both new and retransmissions) arriving at a rate of 30 per second. Packets take 100 ms to transmit.
a) What is G (packets per slot)?
b) What is the probability of success of during a slot?
c) What is the average number of slots per successful transmission?

Problem 6: A communications satellite company establishes a direct connection between a remote town and the company's central office for providing telephone services. Calls arrive as a Poisson process at a rate of 1 per minute. Call durations are Exponentially distributed with an average of 3 minutes. How many circuits should the company provide to ensure that a blocking probability of less than $1 \%$ is maintained?

