16.36 Communication Systems Engineering
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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Aeronautics and Astronautics

16.36: Comm. Sys. Engineering
Problem Set No. 6

Problem 1: Text problem 7.9

Problem 2: Text problem 7.10
Solve for a probability of error of $P_b = 10^{-5}$. The Q function table is located in table 4.1 in the textbook.

Problem 3: Text problem 7.13
Hint: see example 7.5.3 in the textbook for solving the optimal MAP detector.

Problem 4: Text problem 7.60

Problem 5: Matlab Exercise

In this exercise, you will modify your PAM demodulator to be an optimum MAP detector for antipodal signaling, test your demodulator in the presence of noise, and compare the results to the theoretical error probability. We will consider a binary source with p(0) = p, and p(1) = 1-p.

A) Two functions have been uploaded to Stellar for your use in this exercise (you can change them as you like, or make your own)
   a. `rand_bitstring`: Creates a random bit string of size n with bit distribution p(0) = p and p(1) = 1-p, with $p \in [0,1]$.
   b. `error_check`: Takes two bit strings and returns the number of bit errors and the error rate.

B) Modify your PAM demodulator function to use the optimum MAP detector decision point for an antipodal signal.
   a. The optimal MAP detector for arbitrary bit distributions p and 1-p is solved in example 7.5.3 in the textbook.
   b. You will have to add additional inputs to your function to tell it the variance of the noise, σ^2, and the symbol probabilities, p.

C) After you modulate your bit string, you will need to add noise to it
   a. Use the function `randn` to generate Gaussian noise with zero-mean and variance $\sigma^2 = 1$. To change the variance, multiply the output of `randn` by the desired standard deviation, σ.
   b. Create a noise vector with the same number of samples as your signal, which you can add to your modulated signal.
   c. After you add noise to your modulated signal, you can demodulate it using your matched filter demodulator.

D) You can calculate the theoretical error probability using the Matlab function `qfunc`. Since you know the amplitude A, symbol length T, and noise with variance σ^2, you can easily calculate the theoretical value.

E) You will conduct two separate tests
   a. Test 1: for a given distance between constellation points d, add noise with σ ranging from 0 to 5, increasing in 0.2 increments. Plot your observed error rate, as well as the plot of the theoretical error rate from the Q function. Run this simulation for d = 2. Plot the simulation and the theoretical error probability on a single plot and label each line. Use even bit distribution, p = 0.5. Notice how the MAP detector becomes a ML detector for p = 0.5.
   b. Test 2: Run the simulation for different values of p. Same as before, add noise with σ ranging from 0 to 5, increasing in 0.2 increments. In addition, plot the theoretical error probability for only p = 0.5, which is the same as in Test 1, and corresponds to the ML detector. Run this simulation for p = .05, .25, .5, .75, and .95. Put all on a single plot, as well as the ML theoretical error probability, and label the results. You can now see the improvement in performance from the ML detector.

F) Use the following inputs for your tests
   a. Carrier frequency, fc, of 1 Hz.
   b. Sampling frequency, fs, of 4 Hz.
   c. Symbol rate, Rs, of 1.
   d. Bit string size of n = 5000. This is important to give a large enough sample size.
   e. M will now be held constant at 2.

G) Please produce as output the two plots for Test 1 and Test 2. Make sure to comment all of your code and clearly label your plots.