G–β Locus Relations

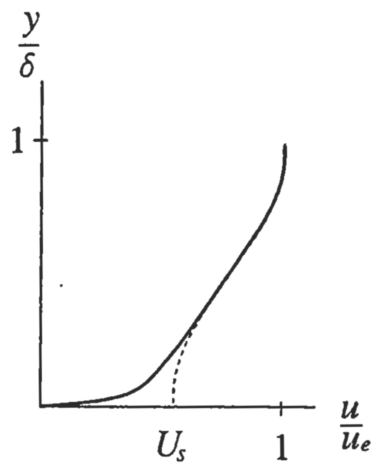
Definitions

$$G \equiv \frac{\int \left(\frac{u_e - u}{u_\tau}\right)^2 dy}{\int \frac{u_e - u}{u_\tau}\, dy} = \frac{1}{\sqrt{C_f/2}}\,\frac{H-1}{H} \qquad , \qquad \beta \equiv \frac{\delta^*}{\tau_w}\frac{dp}{dx} = \frac{-1}{C_f/2}\frac{\delta^*}{u_e}\frac{du_e}{dx}$$

Coles outer-profile relations

$$\frac{u}{u_e} = U_s + (1 - U_s)\left[\frac{1}{2} - \frac{1}{2}\cos\left(\pi\frac{y}{\delta}\right)\right]$$

$$\frac{\delta^*}{\delta} = \frac{1-U_s}{2} \qquad , \qquad \frac{\theta}{\delta} = \frac{1-U_s}{2} - \frac{3}{8}(1-U_s)^2$$

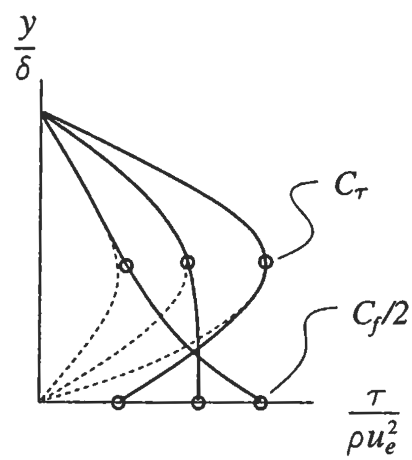


H–U_s relation: $\quad \dfrac{H-1}{H} = \dfrac{3}{4}(1 - U_s)$

eddy viscosity: $\quad \nu_t = K\, u_e\, \delta^* = K\, u_e\, \delta\, \dfrac{1-U_s}{2}$

shear stress: $\quad C_\tau \equiv \dfrac{\tau_{\max}}{\rho u_e^2} = \dfrac{\nu_t}{u_e^2}\left.\dfrac{\partial u}{\partial y}\right|_{y=\delta/2} = \dfrac{\pi}{4} K\,(1-U_s)^2$

Outer-layer/Wall-layer shear stress relation



Using $\quad \left.\dfrac{\partial \tau}{\partial y}\right|_{y=0} = \dfrac{dp}{dx} = -\rho u_e \dfrac{du_e}{dx} \quad \ldots \quad \dfrac{\partial(\tau/\rho u_e^2)}{\partial(y/\delta)} = \beta\,\dfrac{C_f/2}{\delta^*/\delta}$

estimate: $\quad C_\tau \simeq \dfrac{C_f}{2} + \dfrac{\partial(\tau/\rho u_e^2)}{\partial(y/\delta)}\,\dfrac{y_{\max}}{\delta} = \dfrac{C_f}{2}\left(1 + \dfrac{y_{\max}}{\delta^*}\beta\right)$

$$G^2 \equiv \frac{1}{C_f/2}\left(\frac{H-1}{H}\right)^2 \simeq \frac{(3/4)^2(1-U_s)^2}{C_f/2} = \frac{9}{4\pi K}\,\frac{C_\tau}{C_f/2}$$

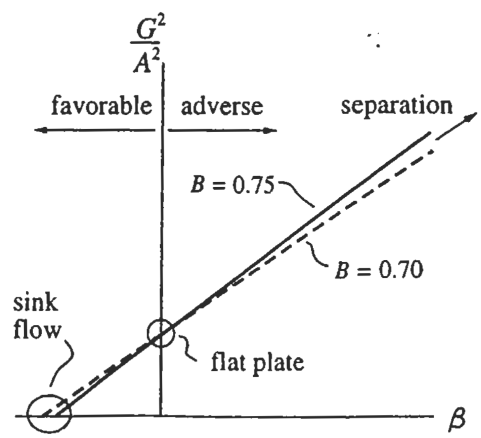
$$\frac{4\pi K}{9}G^2 \simeq 1 + \frac{y_{\max}}{\delta^*}\beta \quad , \quad \text{suggests} \ldots$$

Clauser's Equilibrium Flows (G, β constant)

Empirical fit: $\quad \dfrac{G^2}{A^2} = 1 + B\,\beta \quad$ (G–β Locus)

$A \simeq \dfrac{3}{2}\dfrac{1}{\sqrt{\pi K}}$ = 6.7 Drela ($K = 0.0160$)
= 6.935 Boeing ($K = 0.0149$)
= 6.53 Cebeci-Smith ($K = 0.0168$)

$B \simeq \dfrac{y_{\max}}{\delta^*}$ = 0.75 Drela
= 0.70 Boeing



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