16.06 Principles of Automatic Control
Lecture 31

Example (continued)

We will use the third method. The zero and pole are at

$$s = -4.2 \;\Rightarrow\; z = \frac{1 + sT/2}{1 - sT/2} = 0.9002$$

(lots of digits OK since near z=1.)

$$s = -24 \;\Rightarrow\; z = 0.5385$$

Therefore,

$$K_d(z) = k' \, \frac{z - 0.9002}{z - 0.5385}$$

The determine k', match K and K_d at convenient point. Usually use

$$\begin{aligned} & s = 0, \quad z = 1 \\ \text{or} \quad & s = -2/T, \quad z = 0 \\ \text{or} \quad & s = \infty, \quad z = -1 \end{aligned}$$

In our case

$$K(0) = 42.16 = K_d(1) = 0.216k'$$

$$\Rightarrow \; k' = 195$$

Therefore,

$$K_d(z) = 195 \, \frac{z - 0.9002}{z - 0.5385}$$

which agrees with the Matlab *c2d* command (using 'tustin' option).

How well does controller work?

Must first determine $G_d(z)$. Assuming no processing delay, can find G_d using Matlab:

```
gd=c2d(g,0.025,'zoh')
```

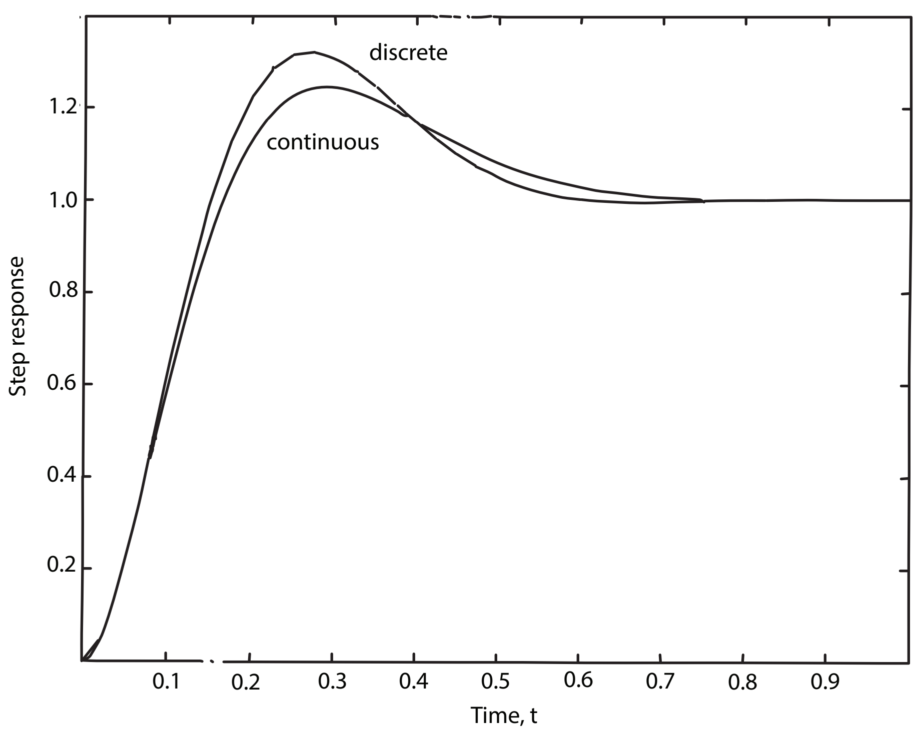
which yields

$$G_d(z) = 0.0003099 \, \frac{z + 0.9917}{(z-1)(z-0.9753)}$$

Can use Matlab to compare step responses for continuous and discrete systems (next page). Note that

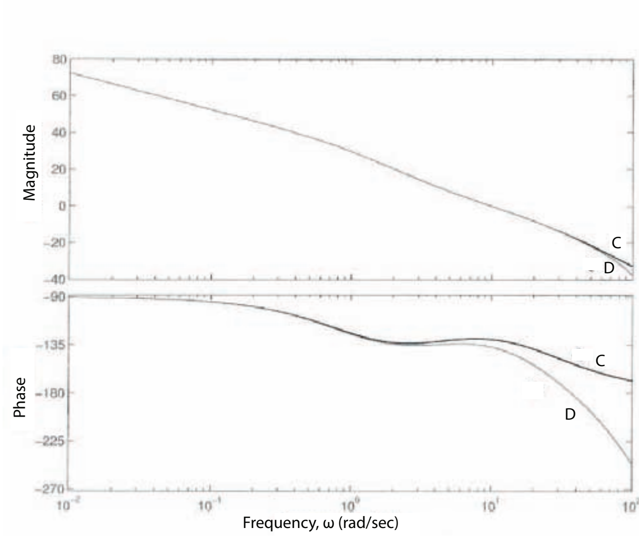
$$M_p = 0.246 \quad \text{(cont.)}$$
$$M_p = 0.321 \quad \text{(disc.)}$$

Why is discrete worse?

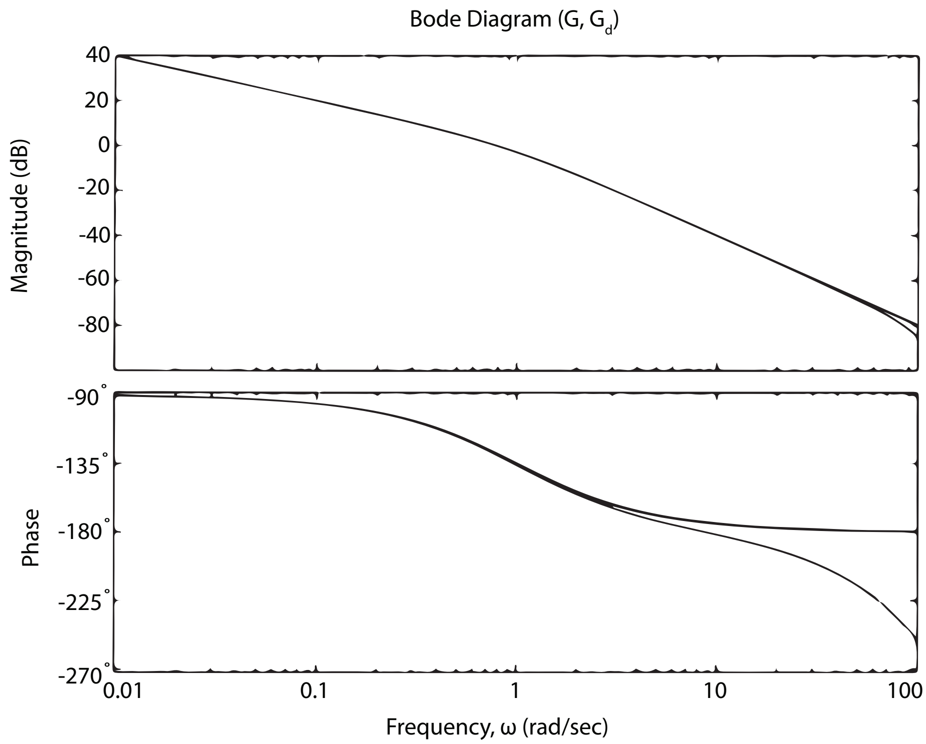


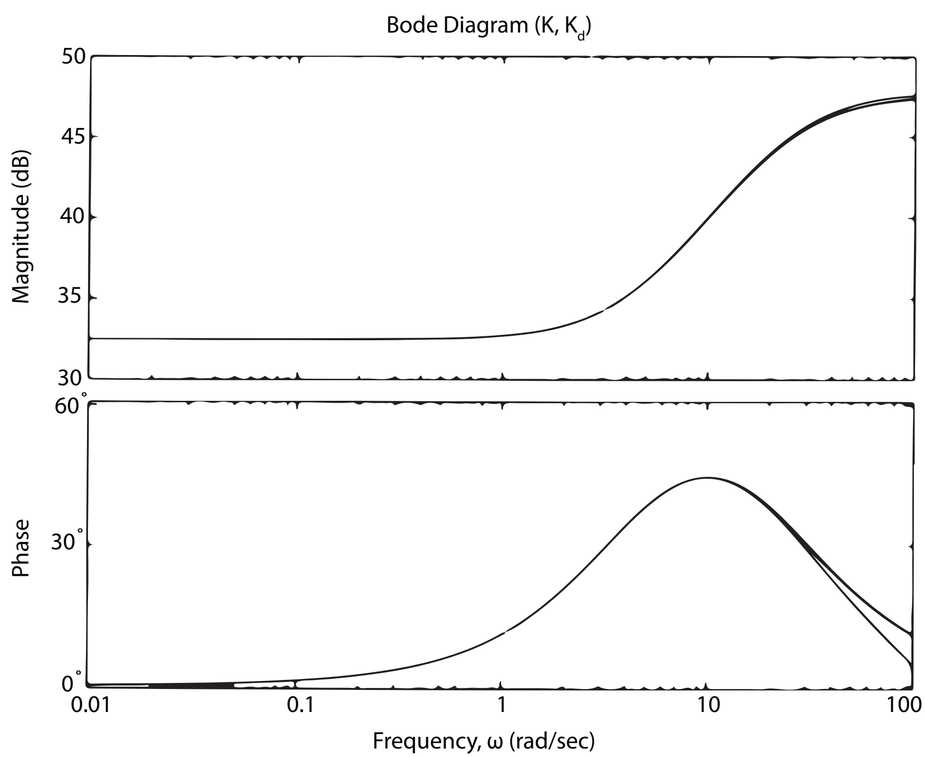
Look at Bode plots of GK, $G_d K_d$ below. The magnitude plots agree well out to 50 r/s, well beyond $\omega_c = 10$ r/s. However, the phase plots are significantly different, even at ω_c. At ω_c, the difference in the phases is $7.2°$. This additional phase lag completely accounts for the increased overshoot.

To see where the phase comes from, look at Bode plot of G, G_d and K, K_d separately. Note that at $\omega_c = 10$ r/s, almost all the additional phase comes from the discretization of G_1, not K.[1]

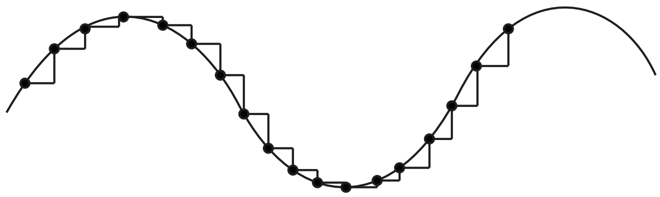


[1] In fact, in some sense the discretization of K produces *no error* at all if prewarping of frequencies is considered.


Bode Diagram (G, G_d)
Magnitude (dB)
40
20
0
-20
-40
-60
-80
Phase
-90°
-135°
-180°
-225°
-270°
0.01
0.1
1
10
100
Frequency, ω (rad/sec)


Bode Diagram (K, K_d)
Magnitude (dB)
50
45
40
35
30
Phase
60°
30°
0°
0.01
0.1
1
10
100
Frequency, ω (rad/sec)

The phase lag in G_d is due to the effect of the zero order hold. To see why, consider sampling then holding a sinusoidal signal:



Note that the reconstructed signal is delayed by about 1/2 the sample period, so there is an additional phase lag of $\omega T/2$. At crossover, this lag is

$$10 \cdot \frac{0.025}{2} \cdot \frac{180^\circ}{\pi} = 7.2^\circ$$

This is *precisely* the additional phase lag seen in the Bode plots!

Compensator Redesign

Since we (now) understand that the ZOH adds phase lag at crossover, we should incorporate that lag from the beginning when we do discrete design. Let's do that now:

$$\angle G(j10) = -174.3^\circ$$

To get PM $= 50^\circ$, need

$$\begin{aligned}\angle K(j10) =& -\angle G(j10) - 180^\circ + \text{PM} + 7.2^\circ \\ =& 51.5^\circ\end{aligned}$$

where $7.2^\circ = \frac{\omega_c T}{2} \cdot \frac{180}{\pi}$ for this case.

Using a centered lead compensator, we get

$$\sqrt{\frac{b}{a}} = 2.86$$

So take

$$\begin{aligned}b =& 10 \cdot 2.86 = 28.6 \approx 29 \\ a =& 10/2.86 = 3.49 \approx 3.5\end{aligned}$$

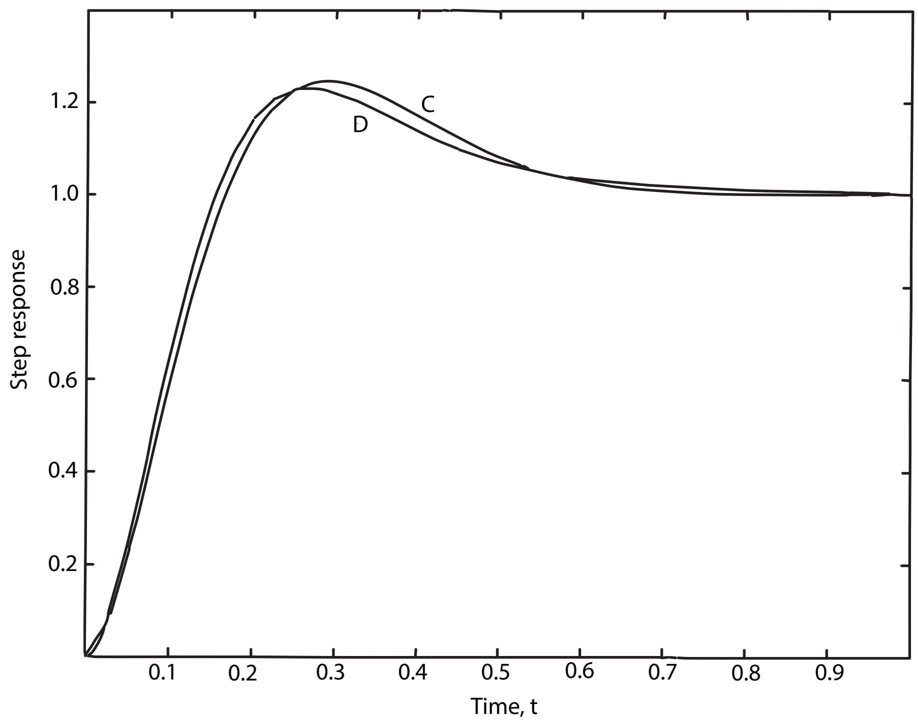
Must also choose gain, so that crossover is at $\omega_c = 10$. Net result is

$$K(s) = 35.12 \, \frac{1 + s/3.5}{1 + s/29}$$

Convert to discrete time by hand or by Matlab *c2d* to obtain

$$K_d(z) = 222.9 \, \frac{z - 0.9162}{z - 0.4679}$$

See step response below.



16.06 Principles of Automatic Control
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