Guided Study Program in System Dynamics
System Dynamics in Education Project
System Dynamics Group
MIT Sloan School of Management[1]

Solutions to Assignment #8
November 15, 1998

Reading Assignment:

Please refer to Road Maps 4: A Guide to Learning System Dynamics (D-4504-4) and read the following papers from Road Maps 4:

- *Generic Structures: First-Order Positive Feedback, by Stephanie Albin and Mark Choudhari (D-4474-1)*
- *Generic Structures: First-Order Negative Feedback, by Stephanie Albin (D-4475-1)*

Please read the following paper:

Mistakes and Misunderstandings: Time Constants, by Kevin Stange (D-4768)

Exercises:

1. Generic Structures: First-Order Positive Feedback

After reading this paper and doing all the included exercises, read the following system descriptions and answer questions below:

You are a drug rehabilitation counselor who is given the choice of two school districts to work in. Both districts show initial stages of drug abuse in the student body. Being a concerned and brave soul, you would like to work for the district that will have the greater number of drug-abusers in ten years if you are not present. The two districts you are presented with are Adams and Barton. Through interviews with school administrators, you have identified the following data:

Adams estimates that about 200 high-school students are active drug users. The number of drug users steadily increases because each drug user tends to influence his or her friends to try drugs. Because the student body as a whole comes from wealthy families,

cash is not a problem. As a result, every drug user influences approximately 0.2 students to become drug users per year. Without you, the rehabilitation counselor, the school does not expect the number of drug users to decrease.

Barton estimates that about 400 students are currently using drugs in the four grades. Fortunately, because the district is comprised mostly of students from lower-income families with budget constraints, every drug user influences only 0.1 additional drug users. Again, the school does not expect the number of drug users to decrease without your aid.

Both school districts are extremely large, so you can assume that there is no market saturation effect.

A. Calculate by hand which high school you expect to have more drug users in ten years. Explain your calculations in your assignment solutions document.
Hint: Think about the doubling time of each system.

The doubling time for a first-order positive-feedback system is
> 0.7 / time constant

The drug-user system fits nicely into a first-order positive-feedback generic structure. Therefore, one can calculate that:

doubling time for Adams = 0.7 / 0.2 per year = 3.5 years and
doubling time for Barton = 0.7 / 0.1 per year = 7 years

Notice, however, that the two schools do not start out with the same initial value of drug users. In fact, Barton's initial number of drug users is double that of Adams. At this point, one cannot tell which school will have more drug users in ten years because even though the number of drug users at Adams doubles faster, Adams starts out with a smaller stock than Barton. The calculations must be based upon both the initial value and the doubling time to get the number of drug users at both schools in 10 years.

Adams:

Years	0	3.5	7	10.5
Drug Users	200	400	800	1600

Barton:

| Years	0	7	14
Drug Users	400	800	1600

In 7 years, both school districts will have 800 drug users. During the 3 years remaining before the tenth year, Barton can expect to gain fewer drug users than Adams. By year 10.5, Adams will see 1600 drug users, while Barton will see the same level only after 14 years. The doubling time of the number of drug users at Adams is shorter than the doubling time of the number of drug users at Barton, so the number of drug users at

Adams is growing faster than the number of drug users at Barton. Hence, by the 10th year, Adams will have many more drug users than Barton.

B. Build a model of the system and use the different parameter values to study the number of drug users in each school district. In your assignment solutions document, include the model diagram, documented equations, and graphs of model behavior. Did the model generate results that you expected? Why or why not?

The model fits the generic structure presented in the paper:

Model diagram:

INITIAL NUMBER OF USERS

new users

Drug Users

INFLUENCE
FRACTION
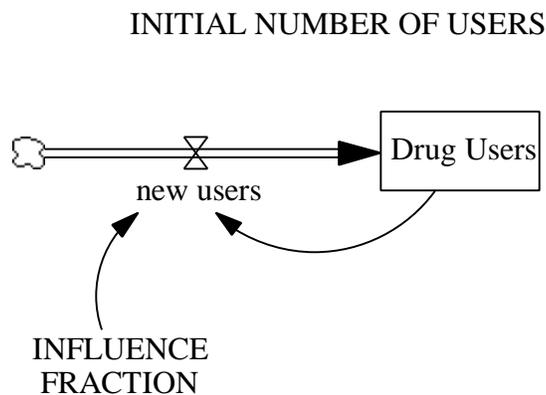
Model equations:

For Adams:

Drug Users = INTEG (new users, INITIAL NUMBER OF USERS)
        Units: user
        The number of drug users in the school district.

INFLUENCE FRACTION = 0.2
        Units: 1/year
        The amount of influence that current drug users have on converting others to drug use.

INITIAL NUMBER OF USERS = 200
        Units: user
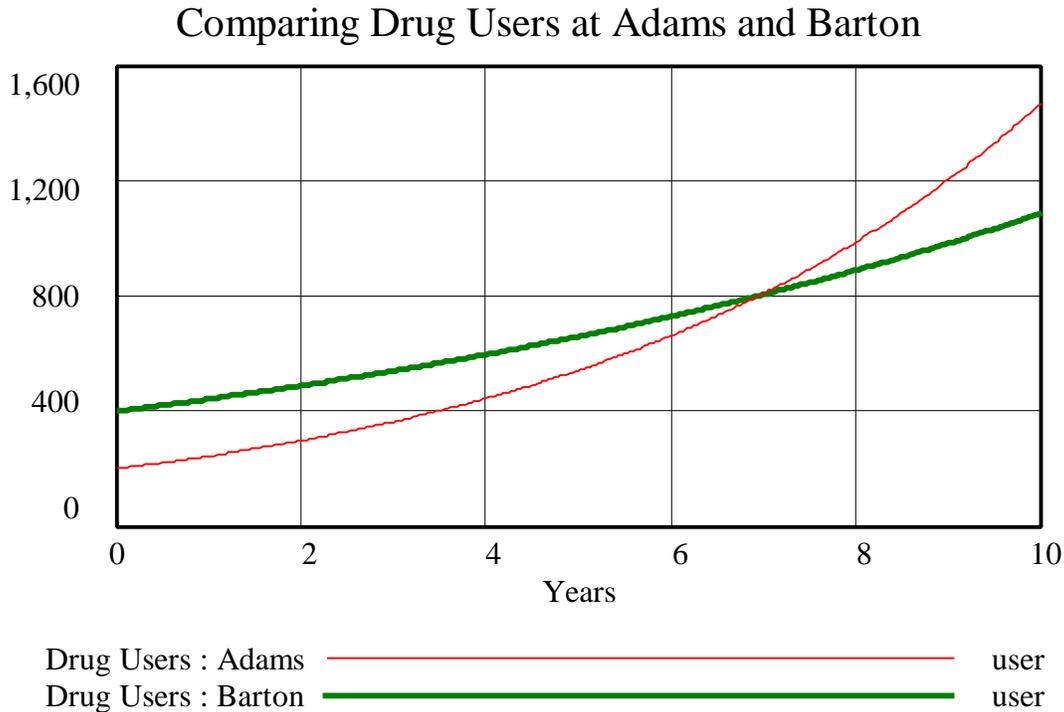   The initial number of drug users in the school district.

new users = Drug Users * INFLUENCE FRACTION
        Units: user/year
        The number of students who become drug users each year.

For Barton, the "INITIAL NUMBER OF USERS" is 400 users and the "INFLUENCE FRACTION" is 0.1 per year.

Simulated over ten years, the model generates the following behaviors for the two school districts:

Comparing Drug Users at Adams and Barton

| Drug Users : Adams | ——————————— | user |
| Drug Users : Barton | ——————————— | user |

The graph of stock behavior confirms the calculations from part A: the number of "Drug Users" at Adams surpasses the number of Barton "Drug Users" around year 7, so Adams has more "Drug Users" in ten years.

C. In addition to your work, the school administration at Adams is considering inviting a speaker to address the students on the dangers of drug abuse once a year. Administrators predict that the speaker will be able to really reach out to a small percentage of the students and be able to decrease influence to about 0.15 - 0.18 new drug users per current drug user. How does this proposal affect your decision of where to work? Please submit graphs for all relevant simulations you used in attempting to answer this question.

This exercise asked you to perform sensitivity analysis on the constant "INFLUENCE FRACTION." Change the constant to values between 0.15 and 0.18, simulate the model, and see how the behavior changes. You should observe that the counselor's decision on where to work really depends on the exact value of "INFLUENCE FRACTION."   The

graph below shows two simulations with the "INFLUENCE FRACTION" set to 0.15 and 0.18 per year, compared with the Barton scenario simulation:

Influence fraction 0.15 and 0.18 for Adams, compared with Barton

Drug Users : Adams15		user
Drug Users : Barton		user
Drug Users : Adams18		user
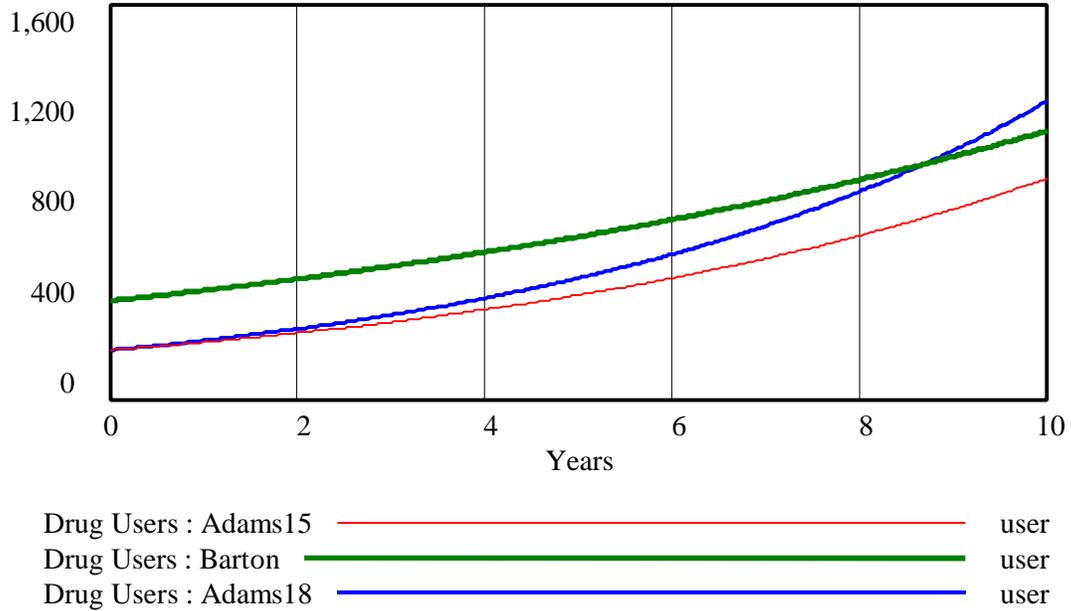
By comparing the above values of "Drug Users" for Adams with the results for Barton, one can see that a "INFLUENCE FRACTION" of 0.l5 puts Adams distinctly below Barton in 10 years, while a "INFLUENCE FRACTION" of 0.18 leaves Adams distinctly above Barton in 10 years. Therefore, the value of "INFLUENCE FRACTION" at which both school districts end up with the same number of users must be somewhere between 0.15 and 0.18. The graph below compares the Barton behavior with the behavior at Adams with "INFLUENCE FRACTION" set to 0.16 and 0.17 per year:

Influence fraction 0.16 and 0.17 for Adams, compared with Barton

Drug Users : Adams16 ————————————	user
Drug Users : Barton ————————————	user
Drug Users : Adams17 ————————————	user
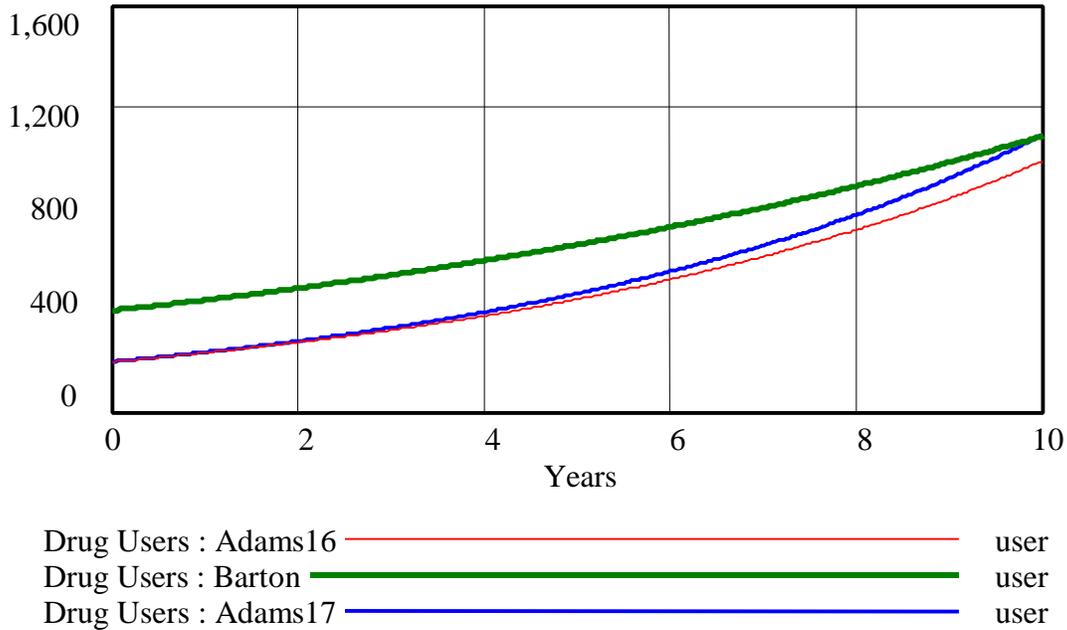
The graph shows that if the Adams "INFLUENCE FRACTION" is 0.16 per year, Adams ends up with a smaller number of "Drug Users" than Barton in 10 years. If the "INFLUENCE FRACTION" at Adams is 0.17 per year, however, the two school districts have approximately the same number of "Drug Users" in 10 years, and you, the counselor, can choose to work at either one.

2. Generic Structures: First-Order Negative Feedback

After reading this paper and doing all the included exercises, read the following system description and answer the questions below:

You are the fund-raising director of Central Park in New York City. Whenever you have a particular project in mind, you submit a quarterly funding request to the city Office of Management and Budget, and usually, about 50% of your request is granted (the city is tight-fisted!). You have just received an excellent architectural proposal for a botanical garden and green house in the middle of the park that will cost about $1 million to build. Remember, when you submit a request you must attach the architect's estimated cost statement and request only the difference between the total cost and the amount already granted, so you cannot make requests for absurd amounts of money. For this problem, you should not take interest rates or discounting into consideration.

Please note that we are again assuming that the events in the system are continuous and not discrete. The project does not receive a bundle of money at the beginning of each quarter, but receives a continuous flow of money, as the Office of Management and

Budget pays, for example, salaries to the construction workers after every instant of work.

A. Build and simulate a model to study how much funding you will request in each quarter for building the garden. In your assignment solutions document, include the model diagram, documented equations, and graphs of model behavior.

The system fits the generic structure of a goal-seeking negative-feedback system. One can model the system in two different ways. The first model uses an inflow goal-gap structure:

Model diagram:
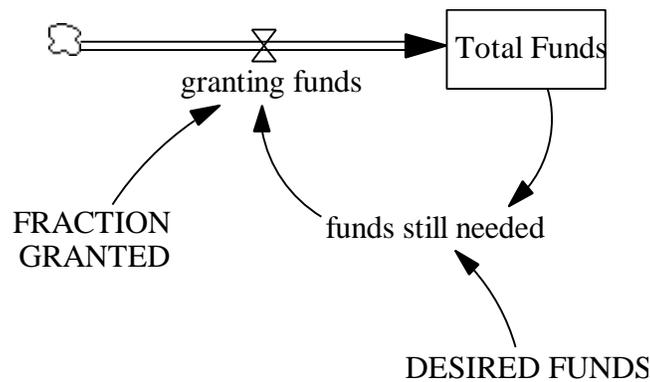
Model equations:

DESIRED FUNDS = 1e+006
    Units: dollar
    The total amount of money needed for the project.

FRACTION GRANTED = 0.5
    Units: 1/Quarter
    The fraction of the needed funds that is granted by the Office of Management and Budget per quarter.

funds still needed = DESIRED FUNDS - Total Funds
    Units: dollar
    The difference between desired and current funds for the project is the amount that still remains to be collected.

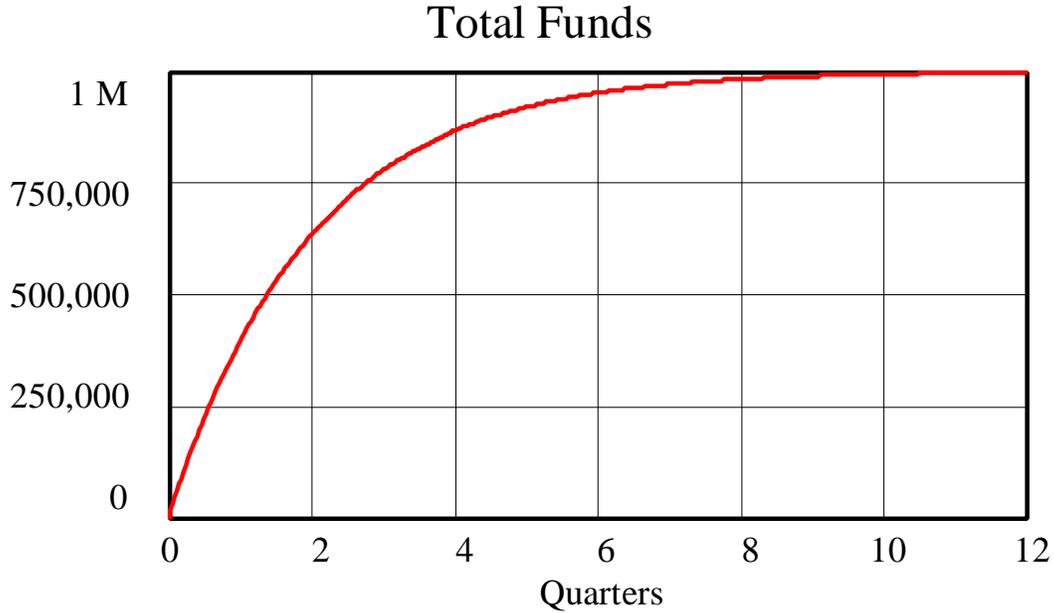granting funds = funds still needed * FRACTION GRANTED
    Units: dollar/Quarter
    The amount of money the Office of Management and Budget grants per quarter.

Total Funds = INTEG granting funds, 0)
    Units: dollar

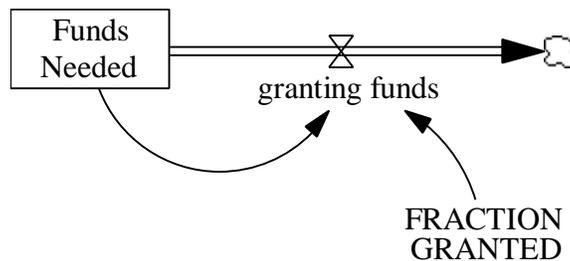The amount of money that has been granted for the project at any given time.

Model behavior:

Total Funds

Total Funds : Funds - inflow ——————————————— dollar

Alternatively, one could model he system with an outflow decay structure:

Model diagram:

Model equations:

FRACTION GRANTED = 0.5
      Units: 1/Quarter
      The fraction of the needed funds that is granted by the Office of Management and
      Budget per quarter.

Funds Needed = INTEG (-granting funds, 1e+006)
      Units: dollar

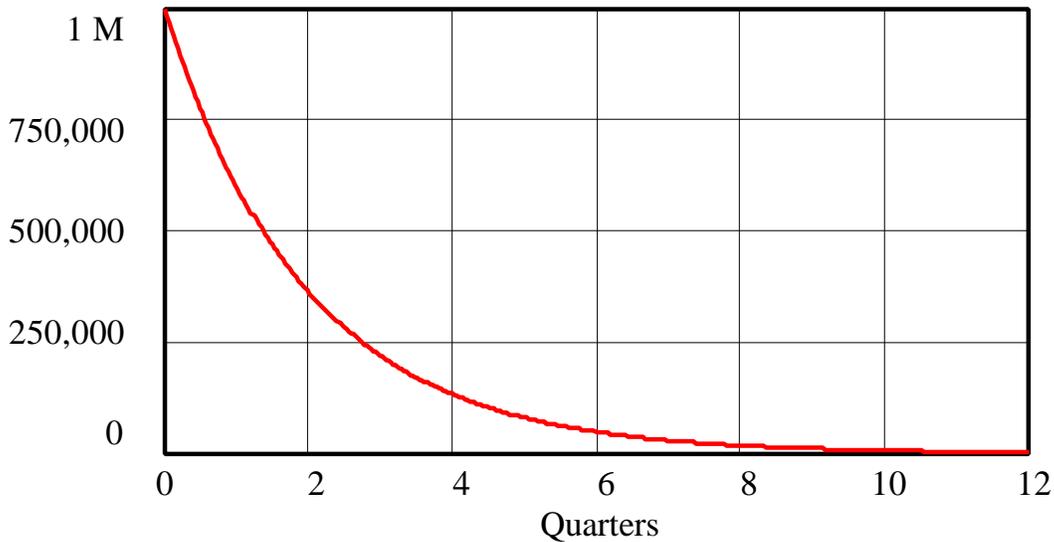The amount of funds that still needs to be collected.

granting funds = Funds Needed * FRACTION GRANTED
        Units: dollar/Quarter
        The amount of money the Office of Management and Budget grants per quarter.

Model behavior:

Funds Needed

Funds Needed : Funds - outflow ———————————— dollar

B. How many fiscal quarters will pass before you are granted 95% of the funds needed for the park (assume you can raise the remaining 5% from private donors once you have the other 95%)?

The time constant of the system is 1 / 0.5 = 2 quarters, so the half-life is 1.4 quarters. Ninety-five percent of the goal of "DESIRED FUNDS" is met when "Total Funds" reach $950,000 and "funds still needed" fall to $50,000. This point occurs after 3 time constants, or after 6 quarters. You can either obtain this value exactly by using exponentials arithmetic or by approximating from the behavior graph. You can also make a table in Vensim that keeps track of the value of "Total Funds" at every incremental period of time. If you make the table, pinpoint the time at which "Total Funds" passes the critical $950,000 mark to find out how many fiscal quarters have passed.

C. Before you submit your first funding request, you suddenly receive an anonymous gift of $500,000 to be used for the botanical garden project. Model this event into the model

from part A. How does this gift affect the number of quarters that will pass before you are granted 95% of the funds needed for the park?

With the anonymous gift, the time constant is still equal to 2 quarters, so the half-life does not change either. If a gift of $500,000 is received at time = 0, then the initial value of "Total Funds," as well as the initial value of "Funds Needed," is $500,000. When there was no gift, it took the system one half-life to reach $500,000. The anonymous gift thus cuts one half-life from the total amount of time that will pass before 95% of the funds have been granted. The number of quarters it takes to reach 95% of the goal is then 6 quarters – 1.4 quarters = 4.6 quarters.

3. Mistakes and Misunderstandings: Time Constants

Please read this paper carefully. You do not have to answer any questions for this paper, but if you can think of an instance when you made the same mistake, feel free to share with us the lesson you learned.