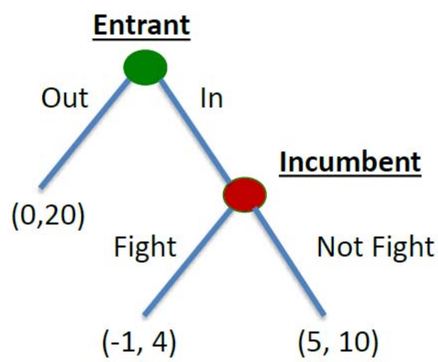
**Problem Set 2 – Solutions**
**Game Theory for Strategic Advantage (15.025)**
**Spring 2015**

1. **Preemptive Investment** Consider the Market Entry game we saw in class: an Entrant must choose whether to go "In" or "Out" of the market. If the Entrant goes In, the Incumbent must choose whether to Fight or Not.

   Choosing In involves a fixed, sunk cost of 5 for the Entrant.

   The following payoffs **do not** include the sunk cost. If the Entrant goes In and the Incumbent fights, they both receive a payoff of 4. If the Incumbent does not fight, they both receive a payoff of 10. If the Entrant stays out, it saves the fixed cost and gets 0, while the Incumbent gets 20.

   (a) *Draw the game tree and the payoffs at each terminal node.*




   (b) *Find the backwards-induction equilibrium.*

   Start from the Incumbent's choice. If the Entrant has chosen In, the Incumbent should Not fight (comparing 20 vs. 10). Therefore, the Entrant should go In expecting a net payoff (subtracting the entry cost) of 10-5=5 if In and 0 if Out.

   (c) *Now imagine that before the game is played, the Incumbent can make an investment. Each investment opportunity entails an up-front cost and modifies the payoffs in the ensuing game. The Incumbent must choose one of the three following options:*

   - *Lobby for tighter regulation. This costs 2 to the incumbent, and increases the Entrant's fixed cost of choosing In from 5 to 8.*

This is a bad idea for the Incumbent. An entry cost of 8 does not deter entry ($10 - 8 = 2 > 0$, for the Entrant); and because nothing else changes in the second stage, the lobbying costs are wasted.

- *Improve its technology so to better tolerate a price war. This costs* 8 *to the Incumbent, and increases its payoff of Fighting from* 4 *to* 12. *(No change to the other payoffs).*

  This is a good idea. Because the cost is paid upfront, were the Entrant to go In, the Incumbent would Fight. This deters entry. Is it profitable? Yes, because the Incumbent ends up with 20-8=12>10 (had it not invested and obtained the backwards induction payoff above).

- *Advertising campaign that shifts* 2 *units of profits from the Entrant to the Incumbent whenever the Entrant chooses In. The campaign costs* 1.

  With the campaign, if the entrant goes in, the incumbent will still Not fight; the entrant's payoff is lower (=3), but still much better than staying out (0). The Incumbent's payoff (net of campaign costs) is then 11.

*Evaluate the three investment opportunities and find the one with the highest return to the Incumbent.*

Overall, the technology improvement is the most expensive but also the most promising choice.

2. **Audition Game** (vaguely based on TV shows) Two venture capitalists want to fund a project. There are two possible projects, and each VC only has resources to fund one. Two entrepreneurs arrive sequentially and pitch their ideas to the VCs (who are sitting in the same room). Each entrepreneur's idea has a net present value $V$ that is uniformly distributed between 0 and 10. VCs don't know the exact value of an idea until they hear the pitch. Upon hearing the first entrepreneur's pitch, both VCs must simultaneously announce "yes" or "no." If only one says yes, she earns the value $V$ of that idea, and leaves the game. If both say yes, a 50:50 coin flip determines which VC gets to fund the entrepreneur, and which one stays in the game. Whoever is left in the game (*i.e.*, one or both VCs) gets to hear the second entrepreneur's pitch, and decide whether to fund it. There is no value for a VC in ending the game without funding a project.

   (a) *Suppose for a moment there was only one VC. Use backwards induction. Which ideas should he fund in the second period if he says "no" to the first one?*

Because the payoff from not funding any project is zero, the VC will say yes to all ideas.

(b) *Consider the single VC's plan of action for the whole game. Fill the blanks. "In the first period, I will fund the following ideas: ______________."*

The expected value of an idea in the second period is 5. The VC will fund them all, so that's the expected payoff if the first idea is turned down. In the first period, the risk-neutral VC should fund all ideas worth 5 and above.

(c) *Now consider the game with two VCs. In order to apply backwards induction, you must compute the following two key values: what is the expected payoff of a VC who enters the second period alone (which occurs if the other VC funds the first idea)? And what is the expected payoff of both VCs entering the second period (which occurs if both say no to the first idea)?*

The expected payoff of entering the second period alone is, as above, equal to 5.

The expected payoff of entering with another VC is 2.5 (because both will say yes and the entrepreneur will choose one at random).

(d) *Suppose the first idea comes in, and both VCs realize it's worth $V = 8$. Find the backwards-induction equilibria of the game.*

(e) *Now suppose the first idea is worth 2. Find the backwards-induction equilibria of the game.*

(f) *Finally, suppose the first idea is worth 4. Find the backwards-induction equilibria of the game.*

If the other VC says yes to an idea worth $V$, my expected payoff is $(1/2)(V+5)$ if I say yes, and 5 if I say no. If the other VC says no, my expected payoff is $V$ if I say yes and 2.5 if I say no.

Therefore, for any $V > 5$, it is a dominant strategy to say yes in the first period.

For any $V < 2.5$, it is a dominant strategy to say no in the first period.

For any $V \in [2.5, 5]$, there are two equilibria of the entire game. One VC says yes in the first period; the other says no; and then funds any idea at round 2.

(g) *Consider each VC's equilibrium plan of action for the whole game. Fill the blanks. "If I ever get to the second idea, I will say _____________; and I will say yes to the first idea if ___________________."*

It is hard to summarize the equilibria described above with a "cutoff" rule. One possible description is "I will say no to $V < 2.5$; yes to $V > 5$ and yes to $V \in [2.5, 5]$ only if my opponent is saying no."

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