

14.472 Spring 2004  
Problem Set 1

Consider a two-period model. In period one everyone works with hours  $y$  and consumption  $c_1$ . In period two everyone is retired with consumption  $c_2$ .

Individual  $h$  has probability  $p^h$  of surviving from period one to period two.

Each worker has expected lifetime utility of  $\alpha \ln(c_1) + p^h \beta \ln(c_2) + \gamma \ln(1 - y)$  where  $\alpha + \beta + \gamma = 1$ .

Assume the wage is  $w$  and the interest rate is  $r$ .

1. Assume there is no annuities market. Derive the optimal lifetime plan for a worker, including expected lifetime utility.
2. Assume there is an actuarially fair annuities market. Derive the optimal lifetime plan for a worker, including expected lifetime utility.
3. Assume there are two workers with different survival probabilities but the same wage and same preferences otherwise. Assume there is an annuities market that breaks even on the market for both of them. Compare the expected lifetime utilities for each with those in your answer to 1. For what values of the survival probabilities do they both gain from the presence of this market?
4. Assume there are two workers with different survival probabilities but the same wage and same preferences otherwise. Assume there is no annuities market. Assume the government introduces a tax at rate  $t$  on earnings. Assume that all of the revenue from the tax (including interest) is used for benefits in period two for surviving workers, with benefits proportional to taxes paid. For what tax rates and survival probabilities are both workers better off than in your answer to 1? Compare lifetime utilities in this case with those in answer to 3.