14.384 Time Series Analysis, Fall 2008
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Supplementary to lectures given by Anna Mikusheva
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Recitation 9

Consumption, Income, Wealth and Cointegration

There is a long line of papers that use cointegration to examine the relationship among consumption, income, and wealth. These notes go over two such papers, one relatively old and one more recent.

Campbell (1987): Does Saving Anticipate Declining Labor Income? An Alternative Test of the Permanent Income Hypothesis

Briefly, the permanent income hypothesis says that consumption should only change in response to changes in expected lifetime income, or "permanent income." Since permanent income is unobservable, the permanent income hypothesis (PIH) cannot be tested directly. Hall (1978) observed that under rational expectations, the PIH implies that consumption should be a martingale ($E[c_{t+1}|\mathcal{I}_t] = c_{t-1}$). Many authors have implemented tests based on this fact, such as Mankiw and Shapiro (1985) as mentioned in lecture 16. Tests based on this fact generally reject the PIH. Campbell develops an alternative test, both to confirm Hall's result and to better understand how the PIH fails.

Model

Let y_{kt} and y_{lt} denote capital and labor income at time t, c_t denote consumption, W_t wealth, and r the interest rate. The PIH model maintains that $y_{kt} = rW_t$. Let's also allow from some unforecastable capital gains, η_t. Then the evolution of wealth is

$$W_t = (1 + r)W_{t-1} + y_{l,t-1} - c_{t-1} + \eta_t$$

We can rewrite the budget constraint in terms of capital and labor income as

$$y_{kt} - (1 + r)y_{k,t-1} - r(y_{l,t-1} - c_t) + \eta_t \tag{1}$$

Consumption is proportional to wealth plus expected future income,

$$c_t = \gamma \left(y_{kt} + \frac{r}{1+r} \sum_{i=0}^{\infty} \frac{E_t y_{l,t+i}}{(1+r)^i} \right) \tag{2}$$

Most of the time, we will assume that $\gamma = 1$. I think we need this for balanced growth anyway. Campbell discusses the possibility of $\gamma < 1$. Let $s_t = y_{tk} + y_{tl} - c_t/\gamma$ and call it savings. Writing (2) in terms of s_t gives,

$$s_t = -\sum_{i=1}^{\infty} \frac{E_t \Delta y_{l,t+i}}{(1+r)^i} \tag{3}$$

We can rearrange to get:

$$s_t - \Delta y_{lt} - (1 + r)s_{t-1} = -r\epsilon_t \tag{4}$$

where $\epsilon_t = \sum_{i=0}^{\infty} \frac{E_t y_{l,t+i} - E_{t-1} y_{l,t+i}}{(1+r)^{i+1}}$ is the change in expected income.

Equations (1) and (4) are two important testable predictions of the PIH.[1] They imply that two linear

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[1] As an exercise you might want to combine these two equations to get Hall's test of the PIH.

combinations of $x_t = [y_{tk}\, y_{tl}\, c_t]$ should not be predictable at time $t-1$. Moreover, if we assume that y_{lt} contains a unit root (an assumption that appears to be true empirically), then we can use results from cointegration to efficiently test these restrictions.

How is cointegration useful? Well, at the start of lecture 16, we saw that we can estimate cointegrating vectors super-consistently, i.e. estimating

$$c_t = \beta_k y_{kt} + \beta_l y_{lt} + e_t$$

results in $\begin{pmatrix} \hat{\beta}_k - \gamma \\ \hat{\beta}_l - \gamma \end{pmatrix} = O_p(1/T)$ instead of the usual $O_p(1/\sqrt{T})$. An implication of this fact is that subsequent regressions involving $\hat{s}_t = y_{tk} + y_{tl} - c_t/\hat{\gamma}$ will have the same asymptotic distribution as if they used the true γ and s_t. (Although, as discussed in class, the finite sample performance could be poor).

Another implication of cointegration is that we must run VECMs instead of VARs to perform our tests. Here's a simple lemma,

Lemma 1. *If each x_t is $CI(1,1)$ i.e. each component of x_t is $I(1)$, but $\alpha' x_t$ is stationary, then there is no invertible MA representation for Δx_t.*

Proof. Exercise. Consider the relationship between the variance of $\alpha' x_t$ and the MA coefficients. $\square$

Engle and Granger (1987) proved that a VECM representation does exist in such a situation. That is,

$$B(L)\Delta x_t = -\gamma \alpha' x_{t-1} + u_t$$

By multiplying by $\begin{pmatrix} \alpha \\ 0 & I \end{pmatrix}$ we can rewrite this as a VAR in $\alpha' x_t$ and some of the components of Δx_t. Since we estimate VARs using OLS equation by equation, we can focus on just a subset of the variables if we want. Consequently, Campbell ultimately just runs bivariate VARs using Δy_{lt} and s_t, and uses the estimates to test the implications of (3) and (4) above.

More specifically, (4) implies that regressing $s_t + \Delta y_{lt} - (1+r)s_{t-1}$ on lags of the variables should result in no significant coefficients. Campbell finds that this restriction is rejected.

Equation (3) implies that s_t should be discounted sum of future Δy_{lt}. Campbell investigates this restriction by calculating $\hat{s}_t = \sum \hat{E}_t \frac{\Delta y_{l,t+s}}{(1+r)^s}$ where $\hat{E}_t$ is the expectation based on the estimated VAR coefficients. He then plots $\hat{s}_t$ (along with standard error bands) and s_t. Using VAR(1), the plots are fairly close. With VAR(5), the variables diverge considerably.

Lettau and Ludvigson (2004) Understanding Trend and Cycle in Asset Values: Reevaluating the Wealth Effect on Consumption

Lettau and Ludvigson use cointegration to separate permanent and transitory shocks to consumption c_t, labor income y_t, and physical wealth a_t (excludes human capital). Using similar reason as Campbell, they manipulate the budget constraint to obtain

$$c_t - \alpha_a a_t - \alpha_y y_t \approx E_t \sum \rho_w^i \left((1-\nu)r_{at+i} - \Delta c_{t+i} + \nu \Delta y_{t+1+i}\right)$$

So if we assume that the intest rate on physical capital, Δc_t and Δy_t are stationary, then c_t, a_t, and y_t are cointegrated with cointegrating vector $\alpha = [1\ -\alpha_a\ -\alpha_y]$. As above, we estimate α superconsistently by using least squares, or if we want to allow for serial correlation, dynamic least squares. From regressions such as this, it is fairly well-established that a permanent one dollar increase in wealth increases consumption by about five cents. Lettau and Ludvigson confirm this finding, and then examine the short-run relationship between transitory changes in wealth and consumption.

As in the previous section let $x_t = [c_t\, a_t\, y_t]$. We can write a VECM for x_t.

$$\Delta x_t = \gamma \hat{\alpha}' x_t + \Gamma(L)\Delta x_{t-1} + e_t$$

We have three non-stationary variables, but a linear combination of the variables is stationary. The variables are generated by three shocks. Interpreting these three shocks has all the same identification problems as VARs. Simple accounting suggests that we should be able to rotate the shocks in a way to identify two that have permanent effects and one that has transitory effects. Indeed, such a decomposition is possible. Gonzalo and Ng (2001) describe how to implement it, and this is what Lettau and Ludvigson do. Lettau and Ludvigson find that most (greater than 90%) of the variation in consumption and labor income is due to permanent shocks, while at least half of the variance of asset wealth is due to transitory shocks. Moreover, they find that transitory shocks have quite long-lasting effects on asset wealth. The half life is about 2 years. They conclude by saying,

> These findings have at least one important implication for monetary policy. Recent research has suggested that central banks pursuing inflation targets should ignore movements in asset values that do not influence aggregate demand (Bernanke and Mark Gertler, 2001). The results in this paper underscore the relevance of this recommendation, since they suggest that most changes in asset values are transitory and unrelated to consumer spending, the largest component of aggregate demand.

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