14.121 Microeconomic Theory I: Waiver Exam

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Instructions. Your grade on this exam does not matter for anything except the decision about whether you need to take (or re-take) the class, so do not panic. You have 90 minutes to complete the exam.

In the following, you are asked to "prove" certain statements. In doing so, you may rely on any results from mathematics you wish, but you should clearly state the steps of your argument and the theorems you reference, if appropriate. Results from economics should be proven unless noted. Partial credit will be given for clear logic and careful reasoning.

1) Consumer Theory

   a) A consumer has a preference relation $\succeq$ that is transitive ($\succeq$ stands for "at least as good as"). Define the strict preference relation $\succ$ and prove that $\succ$ is transitive.

   b) Define the indirect utility function $v(p, w)$ for a consumer. Show that i) v is homogenous of degree 0, ii) strictly increasing in w, iii) nonincreasing in p_l for any l, iv) quasi-convex (that is, the set $\{(p, w)|v(p, w) \leq \bar{v}\}$ is convex for any $\bar{v}$. [You may assume that the consumer's direct utility function u is suitably well behaved.] Why is v nonincreasing rather than strictly decreasing in p_l?

2) Demand Aggregation

   Suppose there are two consumers with utility functions $u_1(x_1, x_2) = x_1 + 4\sqrt{x_2}$ and $u_2(x_1, x_2) = 4\sqrt{x_1} + x_2$. The consumers have identical wealth levels $w_1 = w_2 = w/2$.

   a) Calculate the individual demand functions and the aggregate demand function.

   b) Compute the individual Slutsky matrices $S_i(p, w_i)$ for $i = 1, 2$ and the aggregate Slutsky matrix $S(p, w)$. Does aggregate demand satisfy the weak axiom? Justify your answer.

   c) Let $p_1 = p_2 = 1$. Compute the matrix that is the difference between the sum of the individual Slutsky matrices and the aggregate Slutsky matrix:

   $$C(p, w) = \sum_i S_i(p, w_i) - S(p, w).$$

   What is a sufficient condition on $C(p, w)$ for aggregate demand to satisfy the weak axiom? Is this condition necessary?

3) Consider a pure exchange economy with consumers $i = 1, ..., I$, where consumer i has endowment $e_i > 0$ and locally non-satiated preferences on consumption set $X^i = \mathbb{R}_+^L$. A *coalition* C is a nonempty subset of consumers $\{1, ..., I\}$. It blocks an allocation $\{x_i^*\}_{i=1,...}$ if there exists $x_j \in \mathbb{R}_+^L$, $j \in C$ such that every $j \in C$ strictly prefers x_j to x_j^*, and $\sum_{j \in C} x_j = \sum_{j \in C} e_j$.

a) Prove that a competitive equilibrium allocation cannot be blocked by any coalition, and hence is a *core* allocation.

b) Is every core allocation a competitive equilibrium? Justify your answer with either a proof or counter-example.

4) Consider a pure exchange economy with two goods, $h = 1, 2$ and two consumers, $i = 1, 2$ with utility functions u_1 and u_2 respectively. The total endowment is $e = (e_1, e_2)$ where $e_1, e_2 \gg 0$.

For each of the following cases, determine which of the Pareto-efficient allocations can be decentralized as competitive equilibria with lump sum transfers.

Briefly describe the equilibrium prices and transfers for each Pareto-efficient allocation.

a) $u_1(x, y) = \alpha \ln(x) + (1 - \alpha) \ln(y)$ and $u_2(x, y) = \beta \ln(x) + (1 - \beta) \ln y$, where $\alpha < \beta$ and $\ln x$ is the natural logarithm of x.

b) $u_1 = u_2$ is strictly concave, smooth, and homothetic.

c) For $i = 1, 2$, $u_i(x, y) = x + g(y)$, where g is an increasing and strictly concave function.

d) $u_1(x, y) = \max\{x, 2y\}$, $u_2(x, y) = \max\{2x, y\}$ and $e_1 = e_2$.

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