Lecture 14
Infinitely Repeated Games II

14.12 Game Theory

Muhamet Yildiz

Road Map

1. Folk Theorem
2. Applications (Problems)

Folk Theorem

Definition: $v = (v_1, v_2, \ldots, v_n)$ is feasible iff v is a convex combination of pure-strategy payoff-vectors:

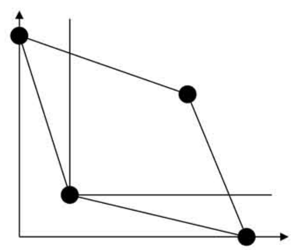
$$v = p_1 u(a^1) + p_2 u(a^2) + \ldots + p_n u(a^m),$$

where $p_1 + p_2 + \ldots + p_m = 1$, and $u(a^j)$ is the payoff vector at strategy profile a^j of the stage game.

Theorem: Let $x = (x_1, x_2, \ldots, x_n)$ be s feasible payoff vector, and $e = (e_1, e_2, \ldots, e_n)$ be a payoff vector at some equilibrium of the stage game such that $x_i > e_i$ for each i. Then, there exist $\underline{\delta} < 1$ and a strategy profile s such that s yields x as the expected average-payoff vector and is a SPE whenever $\delta > \underline{\delta}$.

Folk Theorem in PD

	C	D
C	5,5	0,6
D	6,0	1,1



- A SPE with PV (1.1,1.1)?
  - With PV (1.1,5)?
  - With PV (6,0)?
  - With PV (5.9,0.1)?

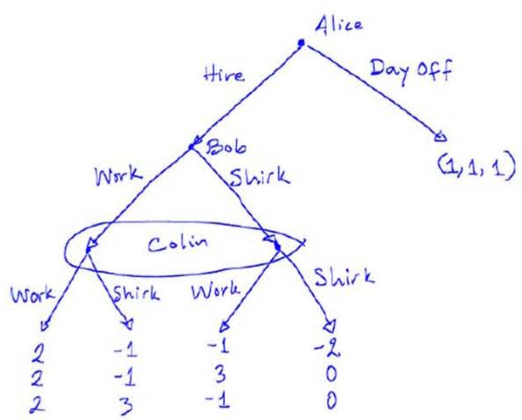
Proof for a special case

- Assume $x = u(a^*) = (u_1(a^*),\ldots, u_n(a^*))$ for some a^*.
- s^*: Every player i plays a_i^* until somebody deviates and plays e_i thereafter.
- Average value of i from s^* is $x_i = u_i(a^*)$.
- s^* is a SPE ⇔ $\delta \geq \bar{\delta}$ where

$$\bar{\delta} = \max_{i,a_i} \frac{u_i\left(a_i, a^*_{-i}\right) - x_i}{u_i\left(a_i, a^*_{-i}\right) - e_i} < 1$$

Applications/Problems

2010 Midterm 2, P2



Range of δ for SPE

- Alice Hires and Bob and Colin both Work until any of the workers Shirk; Alice Hires and Bob and Colin both Shirk thereafter.
- Alice Always Hires. Both workers Work at $t = 0$. At any $t > 0$, each worker Works if the previous play is (Hire, Work, Work) or (Hire, Shirk, Shirk); each worker Shirks otherwise.

2007 Midterm 2, P3

- Stage Game: Linear Bertrand Duopoly (c=0; Q=1-p)
- s*: They both charge 1/2 until somebody deviates; they both charge 0 thereafter.
- s**: n + 1 modes: Collusion, W1, W2, ..., Wn. Game starts at Collusion. Both charge 1/2 in the Collusion mode and p*<1/2 in W1,. . . , Wn. Without deviation, Collusion leads to Collusion, W1 leads to W2,. . . , Wn-1 leads to Wn, and Wn leads to Collusion. Any deviation leads to W1.

14.12 Economic Applications of Game Theory
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